THE PHYSICAL SOCIETY OF LONDON.

PROCEEDINGS.

VOLUME 37.—PART 5.

AUGUST 15, 1925.

Price to Non-Fellows **6s.** *net, post free* **6/3.**

Annual Subscription, **30/-** *post free, payable in advance.*

Published Bi-Monthly from December to August.

LONDON:

FLEETWAY PRESS, LTD.,
3-9, DANE STREET, HIGH HOLBORN, W.C.1.

1925.

THE PHYSICAL SOCIETY OF LONDON.

1925-26.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY OF LONDON

SESSION 1924-1925.

Except where otherwise stated, the meetings were held at the Imperial College of Science, the President being in the chair.

October 24, 1924.

The President announced that owing to a breakdown in health due to overwork Mr. COOPER, the Society's Treasurer, had felt compelled to resign. The Council had accepted the resignation with great regret, and had appointed Mr. R. S. WHIPPLE as Acting Treasurer. The Council proposed to send to Mr. COOPER a letter expressing sincere thanks for his past services to the Society and hopes for his speedy recovery to complete health. With this resolution the Fellows present unanimously associated themselves.

The President also announced that during the vacation Mr. A. V. PARKE, the Society's attendant, had died. It was resolved that a message of sympathy be sent to his widow.

1. "Underblown Pipes," by D. GUNNAIVA, M.A., and G. SUBRAHMANIAM, M.A., with Demonstration by D. J. BLAIKLEY.

2. "On the Temperature Gradient in Gases at Various Pressures," by W. MANDELL, B.Sc., and J. WEST, B.Sc.

3. "Vectorial Dimensions," by J. F. S. ROSS, M.C., B.Sc.

November 14, 1924.

A DEMONSTRATION of an "Electrical Method of Synthesising Vowel Sounds" was given by Dr. W. H. ECCLES, F.R.S.

The following Papers were read :—

1. "The Dielectric Properties of Mica," by D. W. DYE, B.Sc., and L. HARTSHORN, B.Sc., D.I.C., The National Physical Laboratory.

2. "The X-Ray Emission of Electrons from Metal Films, with Special Reference to the Region of the Absorption Limit," by Dr. LEWIS SIMONS, Birkbeck College, London.

November 28, 1924.

A DISCUSSION, arranged jointly with the Royal Meteorological Society, was held on the subject of "Ionization in the Atmosphere, and its Influence on the Propagation of Wireless Signals." Chairman: F. E. SMITH, C.B.E., F.R.S.

PROGRAMME.

1. Opening Address, by Dr. W. H. ECCLES, F.R.S.

2. "Atmospheric Ionization and its Variations," by Dr. C. CHREE, F.R.S.

3. "Experimental Observations of the Strength of Wireless Signals as affected by the Path of the Signals, Time of Day and Season of Year," by Prof. E. V. APPLETON, M.A., D.Sc.

4. "Atmospherics," by R. A. WATSON-WATT.

5. "The Electric Field of a Thunderstorm and Some of its Effects," by C. T. R. WILSON, F.R.S.

6. "The Evidence of Terrestrial Magnetism for the Existence of Highly Ionized Regions in the Upper Atmosphere," by Prof. SYDNEY CHAPMAN, F.R.S.

December 12, 1924.

The following Papers were read :—

1. "Numerical Values of Chemical Constants and Frequencies of the Elements," by Capt. A. C. EGERTON.

2. "The Sensibility of Circular Diaphragms for the Reception of Sounds in Water," by J. H. POWELL, M.Sc.

3. "A Direct-reading Frequency Meter of Long Range," by ALBERT CAMPBELL, B.A.

A DEMONSTRATION of the Frequency Meter was given by W. H. LAWES, of the Cambridge Instrument Co.

January 7 and 8, 1925.

The Annual Exhibition of Apparatus was held by the Physical Society of London and the Optical Society from 3-6 p.m. and from 7-10 p.m. each day.

Discourses were given as follows :—

On January 7 at 4 p.m. and January 8 at 8 p.m., "Some Experiments with Interferometers," by F. TWYMAN, F.R.S.

On January 7 at 8 p.m. and January 8 at 4 p.m., "Talking Motion Pictures," by C. F. ELWELL, B.A., M.I.E.E.

January 23, 1925.

The following Papers were read :—

1. "An Investigation into Corrections Involved in the Measurement of Small Differences in Refractive Index of Dispersive Media by means of the Rayleigh Interferometer, with Special Reference to the Application of the Results to Measurements in Diffusion," by B. W. CLACK, Ph.D., Birkbeck College, London.

2. "A Study of the Production of Flashing in Air Electric Discharge Tubes," by J. TAYLOR, B.Sc., and W. CLARKSON, B.Sc., Armstrong College, Newcastle-on-Tyne.

A DEMONSTRATION entitled "A Kinematographic Study of the Formation of Plateau's Spherule" was given by C. R. DARLING, F.I.C.

February 13, 1925.

Annual General Meeting.

GENERAL BUSINESS.

The Report of the Council and that of the Treasurer were presented and unanimously adopted.

REPORT OF THE COUNCIL.

The chief event of the year has been the celebration of the Jubilee of the Society by various meetings held on March 20 and 21, and a Banquet on March 22. There were large attendances at all these functions, and other Societies were well represented by delegates. At the Jubilee Banquet the Society was honoured by the presence of H.R.H. the Duke of York and the Prime Minister (Mr. J. Ramsay MacDonald). The Society received numerous addresses and messages of congratulation. Details of the celebrations have been published in a special number of the Proceedings.

Eleven ordinary Science Meetings have been held during the year, ten at the Imperial College of Science, and one, on June 27, by kind invitation of Prof. O. W. Richardson, at King's College. In addition, by kind invitation of Sir Ernest Rutherford and the Cambridge Instrument Company, the Society paid on July 19 a visit to Cambridge, where a Science Meeting was held in the Cavendish Laboratory. The visitors spent the morning in the works of the Cambridge Instrument Company, and were entertained to lunch in Hall of Trinity College, by the Directors of the Company.

At the Science Meetings 36 Papers were presented and 13 Demonstrations were given.

On February 14 there was a continuation of the Joint Discussion with the Institution of Electrical Engineers (commenced on November 29, 1923) on "Loud-Speakers for Wireless and Other Purposes." This meeting was held at the Institution and was largely attended.

The Society also co-operated with the Royal Meteorological Society in a Joint Discussion on "Ionization in the Atmosphere and its Influence on the Propagation of Wireless Signals," held at the Imperial College on November 28. Dr. W. H. Eccles opened the Discussion; the attendance numbered about 170.

The Ninth Guthrie Lecture formed part of the Jubilee celebrations at the Institution of Electrical Engineers on March 20. It was delivered by M. le Duc de Broglie on "The Photo-electric Effect in the case of High Frequency Radiation, and some Associated Phenomena." About 250 Fellows and visitors were present.

On January 25 Mr. E. A. Milne gave a lecture to the Society on "Recent Work on Stellar Physics."

Apart from the Joint Discussions and the Guthrie Lecture the average attendance at the meetings of the Society was 66.

The Fourteenth Annual Exhibition of Scientific Apparatus, arranged jointly by the Physical and Optical Societies, was held, through the courtesy of the Governing Body, at the Imperial College on January 2 and 3. Fifty-five firms took part in the Exhibition, which was, as usual, well attended. Discourses were given by Mr. H. B. Grylls on "The Heape and Grylls Rapid Cinema Machine," and by Sir Richard Paget, Bart., on "The Nature and Artificial Production of Human Speech."

At the Centenary celebrations of the Franklin Institute, Philadelphia, in September, Sir William Bragg represented the Society as official delegate and presented on behalf of the Society an Address of Congratulation in the following terms:—

> The Physical Society of London sends greetings to the Franklin Institute on the happy occasion of the hundredth anniversary of its foundation. By the people of Great Britain the name of Franklin has been held in high esteem, both in his lifetime and as a cherished memory. Your Institute during the past hundred years has done much to maintain interest in the broadest aspects of physical science and has thus acted as a monument to the versatility and genius of the man in whose honour the foundation was made. That this most useful work may continue and flourish is the sincere wish of the Physical Society of London.

On behalf of the Society—

F. E. Smith, *President.*

W. R. Cooper, *Treasurer.*

Arthur Schuster, *Foreign Secretary.*

David Owen, A. O. Rankine } *Secretaries.*

The President, Mr. F. E. Smith, and Prof. A. O. Rankine were appointed as delegates to the International Mathematical Congress in Toronto in August.

Mr. F. E. Smith and Dr. D. Owen have been appointed representatives of the Society on the Board of the Institute of Physics, and Mr. T. Smith and Dr. D. Owen on the Science Abstracts Committee. Mr. J. Guild has been appointed, in succession to Mr. T. Smith, on the National Committee for Physics, and Dr. J. Robinson, in succession to Mr. F. E. Smith, on the National Committee for Radio-telegraphy.

At the request of the British Academy Tokyo Committee the Council has presented to the Tokyo University Library Vols. 1 to 27 of the Proceedings to replace losses incurred in the 1923 earthquake.

The Council has invited Prof. A. Fowler, F.R.S., to write a Supplement to his Report on Series in Line Spectra. Prof. Fowler has agreed to do so, and the Supplement, which will reach considerable dimensions owing to the rapid growth of the subject, will be published early in 1925.

The Council has authorised the issue of the Society's Special Reports to Students of the Society at half the published price.

In October, owing to a breakdown in health, Mr. W. R. Cooper, the Treasurer of the Society, was compelled to resign. The Council has accepted the resignation with great regret, and the Society has expressed its sympathy with Mr. Cooper and sincere thanks for his great services to the Society. Mr. R. S. Whipple was appointed Acting Treasurer and co-opted on the Council.

Mr. A. V. Parke, one of the Society's attendants, died suddenly in July. The Society has expressed deep sympathy with the relatives and the Council has subscribed £25 to the fund in aid of the widow.

The Council has awarded the Second Duddell Memorial Medal to Prof. C. Vernon Boys, F.R.S. This Medal will be presented at the Annual General Meeting.

The Society has to record with regret the deaths of Prof. J. D. van der Waals, Prof. G. H. Quincke, Prof. F. D. Brown, Mr. F. Jacob, Mr. J. Grundy, Mr. Chichester A. Bell, Prof. Julius Wertheimer, Dr. R. M. Walmsley and Mr. J. St. V. Pletts. Prof. van der Waals, whose death occurred in 1923 (reference being inadvertently omitted in the 1923 Report), was an Honorary Fellow elected in 1892. So also was Prof. Quincke, the date of his election being 1882. Prof. Brown was a Life Fellow elected very shortly after the foundation of the Society in 1874. Mr. Jacob and Mr. Grundy were also Life Fellows, elected in 1879 and 1884 respectively. Mr. Chichester Bell became a Fellow in 1886. Prof. Wertheimer, Dr. Walmsley and Mr. St. V. Pletts were all Life Fellows, having been elected in 1886, 1888 and 1919 respectively.

The number of Honorary Fellows on the Roll on December 31, 1924, remained at ten, Prof. Max Planck having been elected an Honorary Fellow at the last Annual General Meeting. The number of Ordinary Fellows and Students was 646, this being considerably greater than at the end of 1923. During the year 62 New Fellows and 5 Students have been elected, and there were 10 resignations.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1924.

Expenditure.	£ s. d.	£ s. d.
"Science Abstracts" (Inst. El. Eng.)		281 0 0
Ordinary Publications :—		
"Proceedings"	786 14 2	
"Bulletin"	63 11 5	
General	102 14 2	
		952 19 9
Postage of Publications		87 6 2
Reporting	61 14 4	
Refreshments and Expenses of Ordinary Meetings	26 19 4	
Expenses of Exhibition held jointly with the Optical Society	181 14 0	
Secretary's Expenses	28 8 10	
Treasurer's Expenses	18 14 1	
Periodicals	7 15 0	
Insurance	1 15 0	
Registered Address	2 2 0	
Bank Charges and Cheque Books	0 10 6	
Advertising	4 12 0	
Sundry Expenses	15 12 0	
		349 17 1
Guthrie Lecture (Honorarium)	10 0 0	
Donation to the Institute of Physics towards the Scientific Instrument Journal Guarantee Fund	20 0 0	
Grant to the Widow of A. V. Parke	25 0 0	
		55 0 0
Excess of Income over Expenditure carried down		277 12 8
		£2,003 15 8
Jubilee Celebrations :—		
Expenses at Meetings, Exhibition and Banquet	£293 19 2	
Special Number of the Proceedings	149 13 6	
		£443 12 8

Income.	£ s. d.	£ s. d.
Entrance Fees		46 4 0
Subscriptions by Fellows * (including £70 15 5 Arrears due 1924)	866 5 8	
Subscriptions due from Institute of Physics	10 10 0	
„ Voluntary	9 9 0	
„ by Students	8 18 6	
„ Arrears paid	£46 12 9	
„ Less: Reserved in 1923	42 0 3	
	4 12 6	
„ for "Science Abstracts" and Advance Proofs	36 0 0	
		935 15 8
Sales of Publications (Fleetway Press, Ltd.)		639 17 7
Dividends :—		
London Midland & Scottish Railway	43 7 4	
Metropolitan Board of Works	7 0 0	
Lancaster Corporation Stock	9 6 0	
New South Wales Stock	14 4 11	
Southern Railway	23 9 5	
London & North Eastern Railway	15 10 0	
India 3½% Stock	17 10 0	
Funding Loan	20 3 0	
War Loan 5%	35 0 0	
	185 10 8	
Less: Transferred to Duddell Memorial Trust Fund	20 0 0	
		165 10 8
Income Tax repayable for two years to 31st December, 1924		58 17 3
Fee for the French rights for translating Dr. Jean's report on Radiation		5 0 0
Interest on deposit account		5 5 4
Optical Society for share of Exhibition expenses		8 6 8
Exhibitors' payments on account of Exhibition Catalogue		138 18 6
		£2,003 15 8
Excess of Income over Expenditure brought down	£277 12 8	
Balance carried to Balance Sheet	166 0 0	
		£443 12 8

W. R. COOPER, *Honorary Treasurer.*
ROBERT S. WHIPPLE, *Acting Honorary Treasurer.*

Audited and found correct,
D. DYE
R. L. SMITH-ROSE } *Honorary Auditors.*

February 6th, 1925.

BALANCE SHEET AT DECEMBER 31ST, 1924.

LIABILITIES.

	£ s. d.	£ s. d.
Accumulated Fund:—		
Balance as per last account	1,941 5 4	
Add:—		
Increased value of Investments...£140 0 0		
Less Duddell Memorial Trust Fund £6 0 0		
Less W. F. Stanley Trust Fund 29 0 0		
	35 0 0	
	105 0 0	
	2,046 5 4	
Less Excess of Expenditure over Income	166 0 0	
		1,880 5 4
Life Compositions:—		
Balance as per last Account	2,038 10 0	
Added during year	115 10 0	
		2,154 0 0
Duddell Memorial Trust Fund		432 12 9
W. F. Stanley Trust Fund		449 0 0
Sundry Creditors		149 13 3
Subscriptions paid in advance		41 13 4
		£5,107 4 8

ASSETS.

	£ s. d.	£ s. d.
Investments at Market Value on date:—		
£399 London Midland & Scottish Railway 4 per cent. Debenture Stock	337 0 0	
£1,000 London Midland and Scottish Railway 4 per cent. Preference Stock	801 0 0	
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	189 0 0	
£400 Lancaster Corporation 3 per cent. Redeemable Stock	293 0 0	
£254 2s. 9d. New South Wales 5 per cent. Inscribed Stock, 1935-55	257 0 0	
£300 Southern Railway Preferred Ordinary Stock	253 0 0	
£442 Southern Railway Deferred Ordinary Stock	196 0 0	
£500 London & Eastern Railway 4 per cent. Debenture Stock	417 0 0	
£500 India 3½ per cent. Stock	335 0 0	
£650 4% Funding Loan, 1960-90	580 0 0	
£300 5% War Loan 1929/47 inscribed, "A" Account	304 0 0	
£400 5% War Loan 1929/47 inscribed, "B" Account (Duddell Memorial Fund)	406 0 0	
		4,368 0 0
Stock of Publications (as per Treasurer's valuation)		400 0 0
Debtors—		
Due from Institute of Physics	10 10 0	
" " Optical Society	8 6 8	
" " Arrears of Subscriptions	70 15 5	
" " Customs and Excise for Income Tax reclaim	58 17 3	
		148 9 4
Cash at Bankers	189 7 9	
Cash in hand of Treasurer	1 7 7	
		190 15 4
		£5,107 4 8

W. R. COOPER, *Honorary Treasurer.*
ROBERT S. WHIPPLE, *Acting Honorary Treasurer.*

Audited and found correct,
D. DYE
R. L. SMITH-ROSE } *Honorary Auditors.*

February 6th, 1925.

Note.—The value of the Society's library has not been brought into the Balance Sheet.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1924.

		£	s.	d.
128	Fellows paid £10	1,280	0	0
3	Fellows paid £15	45	0	0
1	Fellow paid £20 10s.	20	10	0
13	Fellows paid £21	273	0	0
17	Fellows paid £31 10s.	535	10	0
162		£2,154	0	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN").

	£	s.	d.
£300 Southern Railway Preferred Ordinary Stock	253	0	0
£442 Southern Railway Deferred Ordinary Stock	196	0	0
	£449	0	0

	£	s.	d.
Carried to Balance Sheet	449	0	0
	£449	0	0

DUDDELL MEMORIAL TRUST FUND.

CAPITAL.

	£	s.	d.
£400 War Loan 5% 1929/47 inscribed Stock	406	0	0

	£	s.	d.
Carried to Balance Sheet	406	0	0

REVENUE.

	£	s.	d.
Balance at 31st December, 1923	26	12	9
Dividends	20	0	0
	£46	12	9

	£	s.	d.
Honorarium to Medallist	20	0	0
Balance carried to Balance Sheet	26	12	9
	£46	12	9

Audited and found correct,

D. DYE
R. L. SMITH-ROSE } *Honorary Auditors.*

W. R. COOPER, *Honorary Treasurer.*
ROBERT S. WHIPPLE, *Acting Honorary Treasurer.*

February 6th, 1925.

Report of the Treasurer.

It has been considered advisable to remodel the method of presenting the accounts so that the financial position of the Society can be more easily ascertained. The Life Composition Fees have been credited to the Life Composition Fund shown on the liability side of the balance-sheet, and not credited to the income and expenditure account, as in previous years. In a similar manner the subscriptions paid in advance have been deleted from the income and expenditure account, and are included as assets in the balance-sheet. The accounts show that the expenditure for the year exceeded the income by £166. An expenditure of £443 12s. 8d. was incurred in connexion with the Jubilee Celebrations of the Society. If this amount is deducted from the expenditure, the excess of income over expenditure amounts to £277 12s. 8d.

Fellows of the Society will be gratified at the increase in the amount received from the sale of the publications of the Society, the sum so received this year amounting to £639 17s. 7d., an increase of £140 9s. 8d. over that received last year.

The investments have been valued at market prices through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank, owing to the rise in value of the securities, the market value of the Society's investments has increased during the twelve months by £140.

Election of Officers and Council.

Dr. Lewis Simons and Dr. B. W. Clack having been appointed Scrutators, a ballot was held for the Officers and Council for the ensuing year. The following were elected :—

President.—F. E. Smith, C.B.E., F.R.S.

Vice-Presidents (who have filled the office of President).—Sir Oliver J. Lodge, D.Sc., F.R.S., Sir Richard Glazebrook, K.C.B., D.Sc., F.R.S., C. Chree, Sc.D., LL.D., F.R.S., Prof. H. L. Callendar, M.A., LL.D., F.R.S., Sir Arthur Schuster, Ph.D., Sc.D., F.R.S., Sir J. J. Thomson, O.M., D.Sc., F.R.S., Prof. C. Vernon Boys, F.R.S., Prof. C. H. Lees, D.Sc., F.R.S., Prof. Sir W. H. Bragg, K.B.E., M.A., F.R.S., Alexander Russell, M.A., D.Sc., F.R.S.

Vice-Presidents.—E. H. Rayner, M.A., Sc.D., J. H. Vincent, D.Sc., M.A., D. Owen, B.A., D.Sc., C. R. Darling, F.I.C.

Secretaries.—Prof. A. O. Rankine, O.B.E., D.Sc., Imperial College of Science and Technology; J. Guild, A.R.C.Sc., D.I.C., National Physical Laboratory, Teddington, Middlesex.

Foreign Secretary.—Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

Treasurer.—R. S. Whipple, 45, Grosvenor Place, S.W.1.

Librarian.—J. H. Brinkworth, M.Sc., A.R.C.Sc., Imperial College of Science and Technology.

Other Members of Council.—R. W. Paul, Prof. A. M. Tyndall, D.Sc., W. S. Tucker, D.Sc., J. S. G. Thomas, D.Sc., D. W. Dye, B.Sc., Prof. F. L. Hopwood, D.Sc., E. A. Owen, B.A., D.Sc., J. Robinson, M.Sc., Ph.D., G. B. Bryan, D.Sc.

Election of an Honorary Fellow.

Professor W. Wien was elected an Honorary Fellow of the Society.

Death of a Distinguished Fellow.

The President announced the death of Mr. Oliver Heaviside, F.R.S. At his suggestion the Fellows present signified, by standing, their regret and their sympathy with Mr. Heaviside's relatives.

Votes of Thanks.

The following votes of thanks were moved: (*a*) To the Auditors (Messrs. D. Dye and R. L. Smith-Rose), moved by Dr. J. S. G. Thomas and seconded by Dr. E. H. Rayner. (*b*) To the retiring Officers and Council (special mention being made of the valuable services of Dr. D. Owen, who retires from the secretaryship), moved by Dr. C. V. Drysdale, O.B.E., and seconded by Mr. W. C. S. Phillips. (*c*) To the Governors of the Imperial College of Science, moved by Mr. T. Smith and seconded by J. Guild.

ORDINARY MEETING FOLLOWING THE ANNUAL GENERAL MEETING.

The President, Mr. F. E. Smith, F.R.S., delivered an address entitled "A System of Electrical Measurements." The address was illustrated by experiments.

February 27, 1925.

Presentation of the Duddell Medal.

The President presented the Duddell Medal to Prof. C. V. Boys, F.R.S., and gave the following account of Prof. Boys' work :—

Prof. Boys is one of the oldest Fellows of the Physical Society, and in the early history of the Society he took a most active part. He has been Demonstrator, Secretary and President of this Society. The Duddell medal is essentially an award for producing apparatus to advance science, and Prof. Boys is renowned not only for his knowledge of physics, but for his exceptional constructive ability. He invented the first method for producing quartz fibres; indeed, it was he who first had the idea of making fibres of very small diameter and freedom of elastic fatigue; he made a most beautiful torsion balance for measuring the Constant of Gravitation; he invented the radio-micrometer—that beautifully sensitive instrument which will measure the heat radiated from a candle at a distance of three miles; he it was who first photographed a bullet in flight, and afterwards showed us how to blow soap bubbles and study the thin films of which they are formed. A little more than a year ago he exhibited a most beautiful calorimeter, and in the future I feel sure he will produce many other beautiful and valuable instruments. All his apparatus is characterized by exceptional beauty of design—as an experimenter he is unexcelled. It is with the greatest pleasure that I make on behalf of the Society the presentation of the Duddell Medal to Prof. Boys.

Prof. Boys, after expressing appreciation of the award, made some remarks as to the ideals he had kept before him in the design of apparatus. It was comparatively easy to increase the sensibility of instruments, if one had no objection to multiplying the experimental errors by a corresponding factor, but he had always made it his aim to attain the highest sensibility while keeping down such errors. Unless such precautions

were taken the strangest phenomena might be observed with highly sensitive apparatus, in consequence, for instance, of convection current disturbances.

The following Papers were read :—

1. "Notes Concerning the Sprengel Pump," by J. J. MANLEY, M.A., Research Fellow of the Magdalen College, Oxford.

2. "The Thomson Effect in Copper, Iron, and Carbon Steels," by J. YOUNG, B.Sc., A.R.C.Sc., F.R.A.S., Lecturer in Physics at the University of Birmingham.

3. "An Improved Cathode-Ray Tube Method for the Harmonic Comparison of Frequencies and for the Delineation of their Wave Form," by D. W. DYE, B.Sc., of the National Physical Laboratory.

A DEMONSTRATION was given in illustration of Paper No. 3.

March 13, 1925.

The following Papers were read :—

1. "The Variation of Young's Modulus at High Temperatures," by JAS. P. ANDREWS, B.Sc., East London College.

2. "The Critical Velocity of Flow Past Objects of Aerofoil Section," by E. G. RICHARDSON, B.A., M.Sc., Ph.D., University College, London.

3. "A Focussing Method of Crystal Powder Analysis by X-Rays," by J. BRENTANO, D.Sc., Lecturer in Physics, Manchester University.

March 27, 1925.

The following Papers were read :—

1. "Electrical Conditions Arising at a Gas-Liquid Interface," by H. W. GILBERT, B.Sc., and P. E. SHAW, B.A., D.Sc.

2. "A Contact Theory of Dielectric Absorption and Power Loss," by L. HARTSHORN, A.R.C.Sc.

The following DEMONSTRATIONS were given by Mr. NORMAN KIPPING, of the International Western Electric Company :—

1. "The Use of the Neon Lamp for Producing an Even Time Base for the Delineation of Wave Forms."

2. "An Examination of Some Dynamic Characteristics of Valves."

3. "A Method of Determining the Percentage Modulation and Reproduction Characteristics of a Radio Transmitting Equipment."

April 24, 1925.

The Tenth Guthrie Lecture was delivered by PROF. W. WIEN, who took as his subject "Recent Researches in Positive Rays."

A vote of thanks to the Lecturer was proposed by Dr. F. W. ASTON, F.R.S., seconded by Dr. W. H. ECCLES, F.R.S., and carried by acclamation.

May 8, 1925.

The following Papers were read :—

1. "A Magnetic Bridge for Testing Straight Specimens and an Analysis of the Hysteresis Loop of Cobalt Chrome Steel," by EDWARD HUGHES, B.Sc.

2. "The Experimental Control of Electrically Broadened Spectral Lines," by M. C. JOHNSON, B.A.

3. "On the Spectra of the Metals of the Aluminium Sub-group," by M. K. RAO.

A DEMONSTRATION entitled "The Diffraction of Light by a Spherical Obstacle" was given by Prof. A. O. RANKINE, O.B.E., D.Sc.

May 22, 1925.

Prof. L. S. ORNSTEIN, of Utrecht, gave a lecture entitled "The Intensity of Spectral Lines—Measurement and Theory."

A vote of thanks to the Lecturer was proposed by Dr. ALEXANDER RUSSELL, F.R.S., and was carried by acclamation.

June 12, 1925.

The following Papers were read :—

1. "On Mass and Energy," by G. TEMPLE, Ph.D., Imperial College of Science."

2. "The Characteristic Curves of Liquid Jets," by E. TYLER, M.Sc., and E. G. RICHARDSON, M.Sc., University College, London.

The following DEMONSTRATIONS were given :—

"Some Experiments to Illustrate the Application of the Shakespeare Katharometer to Physical Research," by Dr. H. L. DAYNES, of the Cambridge Scientific Instrument Co.

"Some Simple Apparatus for the Estimation of Carbon Dioxide," by Dr. EZER GRIFFITHS, of the National Physical Laboratory.

June 26, 1925.

E. H. RAYNER, M.A., Sc.D., in the Chair.

The following Papers were read :—

1. "The Electrostatic Capacity of Two Spheres when Touching One Another," by ALEXANDER RUSSELL, D.Sc., F.R.S., Principal of Faraday House.

2. "An Investigation of the Control Conditions under which Newton's Law is Valid for the Emission of Heat from Electrically Heated Wires," by Miss S. MARSHALL and J. O. C. VICK.

3. "Condensation Nuclei Produced by the Illumination of Air-Halogen Mixtures," by IVOR JONES, B.Sc., Garrod Thomas Research Student, University College. Aberystwyth.

July 4, 1925.

A SPECIAL MEETING was held at Oxford, by kind invitation of Prof. F. A. LINDEMANN, F.R.S., who entertained the Society at lunch in Christ Church Hall and at tea in Wadham College Hall. The meeting was held in the Clarendon Laboratory, the PRESIDENT being in the Chair.

The following Papers were read and illustrated by Demonstrations :—

1. "A Photoelectric Radiation Pyrometer," by Prof. F. A. LINDEMANN, F.R.S., and T. C. KEELEY.

2. "The Determination of Ozone in the Upper Air," by G. M. B. DOBSON and D. N. HARRISON.

3. "The Absorption of Electrons in Metal Vapours," by R. B. BRODE.

4. "Some Mercury Vapour Pumps," by M. W. GARRETT.

5. "Two New Types of Low Pressure Gauges," by E. BOLTON KING.

6. "An Application of Spectrography to the Measurement of High Temperatures," by I. O. GRIFFITH.

7. "X-Ray Analysis of Bent Rocksalt Crystals," by C. H. BOSANQUET.

A cordial vote of thanks to Prof. Lindemann was proposed by the PRESIDENT and carried by acclamation.

XXIII.—ON MASS AND ENERGY.

By G. Temple, *Ph.D.*, Imperial College of Science.

Received April 21, 1925.

Abstract.

It is assumed that variations in the potential energy of a body (gravitational or electrostatic) are always accompanied by proportionate changes in its mass. Combining this assumption with the theories of Newtonian dynamics and Maxwellian electrodynamics, it has been found possible to predict all those phenomena, which are usually regarded as the crucial tests of the theories of relativity, both "special" and "general."

Contents.

§1. Introduction.
§2. The Equivalence of Mass and Energy.
§3. The Secular Advance of the Perihelion of Mercury.
§4. The Deviation of Light-Rays by the Sun.
§5. The Displacement of the Fraunhöfer Lines.
§6. The Variation of Electronic Mass with Velocity.
§7. The Fine-Structure of the Spectrum of H and He^+.
§8. Conclusion.

§1. Introduction.

REGARDED from the standpoint of the experimental physicist, the justification of a theory of relativity is to be found in the fact that it satisfies both of the following conditions. In the first place, to a high degree of approximation, the relativistic theory must reproduce the essential features of classical dynamics and electromagnetism, which are, of course, substantially accurate summaries of experimental facts. In the second place, the slight corrections and modifications made by the relativistic theory must be such as to afford some explanation of those phenomena which have eluded investigation by the older theories. In this Paper we shall discuss a number of phenomena, the explanation of which has been one of the triumphs of the General and Special Theories of Relativity, basing our discussion on an hypothesis which seems to be more congenial to physicists than the elusive metaphysical basis of the theories of Einstein and Whitehead. It will transpire that the simple assumption we shall make, supported by elementary mathematics, is sufficient to account for most of the "crucial" phenomena of relativity.

§2. The Equivalence of Mass and Energy.

In making a slight modification of the classical theories of physics it is an obvious advantage to conserve as far as possible the mathematical form of these theories. To this end we shall adopt in our new theory all the fundamental laws of Newtonian dynamics, making but one alteration—we shall not assume the masses of our moving bodies to be constants, but shall assume them to vary with the potential energies of the bodies, in a fashion to be made precise in the following manner.

Let m denote the mass of a particle, and K denote its potential energy, while c denotes the velocity of light in vacuo and t the time. Then our fundamental assumption is expressed by the equation

$$\frac{d}{dt}\left(\frac{1}{2}mc^2\right)+\lambda\frac{dK}{dt}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}1)$$

where λ equals unity when the energy is gravitational, and equals one-half when the energy is electrostatic. As the potential energy of a particle decreases, there will be a proportionate increase in its mass. The first term in the above equation has a form which recalls an hypothesis of Sir J. J. Thomson[1], according to which all potential energy is kinetic in origin.

Let U denote the potential of a gravitational field of force at any point, and m denote the mass of a particle when placed at the same point. Then

$$K=mU \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}2)$$

and

$$\frac{d}{dt}\left(\frac{1}{2}mc^2\right)+\frac{d}{dt}(mU)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}3)$$

from equations (2·1) and 2·2).

Integrate this equation from $t=t_1$ to $t=t_2$, and let m_1, m_2, U_1, U_2 denote the initial and final masses and potentials respectively. Then

$$\tfrac{1}{2}(m_2-m_1)c^2=-(m_2U_2-m_1U_1)$$

or

$$m_2=\frac{1+\frac{2U_1}{c^2}}{1+\frac{2U_2}{c^2}}\cdot m_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}4)$$

If m_0 denotes the mass of the particle when removed to an infinite distance from the matter producing the field of force, and if we adopt the usual convention that U vanishes at infinity, then we obtain the results

$$m=\frac{m_0}{1+\frac{2U}{c^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}5)$$

and

$$mU=\tfrac{1}{2}m_0c^2-\tfrac{1}{2}mc^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}51)$$

If x, y, z denote the Cartesian co-ordinates of the particle at time t, the Newtonian equations of motion will be

$$\frac{d}{dt}\left\{m\frac{dx}{dt},\ m\frac{dy}{dt},\ m\frac{dz}{dt}\right\}=-\frac{\partial}{\partial x,\ y,\ z}(mU) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}6)$$

We shall transform these equations by taking as our new independent variable the " proper time " τ defined by the integral

$$\tau=\int_0\left(1+\frac{2U}{c^2}\right)dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}7)$$

taken along the path traversed by the particle from the time $t=0$. Noticing that

$$\frac{dt}{d\tau}=\left(1+\frac{2U}{c^2}\right)^{-1}=\frac{m}{m_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}71)$$

we obtain the result

$$m_0\frac{d^2}{d\tau^2}\cdot\{x, y, z\}=-\frac{m}{m_0}\frac{\partial}{\partial x, y, z}(mU)$$

$$=\frac{c^2}{4m_0}\cdot\frac{\partial}{\partial x, y, z}(m^2) \text{ from } (2{\cdot}51)$$

$$=-\frac{m_0\,.\,\partial V}{\partial x, \partial y, \partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}8)$$

where $$V=-\frac{c^2}{4}\left(1+\frac{2U}{c^2}\right)^{-2}\sim-\frac{c^2}{4}+U- \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2{\cdot}81)$$

from (2·71).

Hence the path described by a particle is exactly that which would have been obtained on the classical theory in a field of force of potential V.

§3. The Secular Advance of the Perihelion of Mercury.

An immediate deduction from the conclusion of the preceding paragraph is that the equations to the trajectory of a particle used in Newtonian dynamics may also be utilised in our new theory, provided we take as the potential the function V. For a central field of force due to a massive immovable body of mass M, the true potential is given by the equation

$$U=-\frac{\gamma M}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}1)$$

where γ is the constant of gravitation ($=6{\cdot}66\times10^{-8}$ gms.$^{-1}$. cms.^{3} . secs.$^{-2}$), and r is the radius vector drawn from the centre of the attracting body. Hence, using u to denote the reciprocal of r, we find from equation (2·81) that the "apparent" potential is

$$V=-\frac{c^2}{4}\,.\,(1-2\mu\,.\,u)^{-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}11)$$

where $\mu=\frac{\gamma M}{c^2}=$ the "gravitational radius" of the attracting body. Taking the mass of the sun as $M=1{\cdot}99\times10^{33}$ grams, we find that its gravitational radius is $\mu=1{\cdot}47$ kilometres.

The differential equation to the orbit of a planet may now be written in the familiar form

$$\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}2)$$

where θ is the angular co-ordinate, h is Kepler's constant equal to $r^2\frac{d\theta}{d\tau}$, and P the apparent central acceleration is equal to

$$\frac{\partial V}{\partial r}=\frac{\mu c^2 u^2}{(1-2\mu u)^3}$$

Hence equation (3·2) may be written

$$\frac{d^2u}{d\theta^2}+u=\frac{\mu c^2}{h^2(1-2\mu u)^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}21)$$

which differs from the corresponding equation of the classical theory, in which the term $(1-2\mu u)^3$ is replaced by unity.

A first integral of this equation is

$$\left(\frac{du}{d\theta}\right)^2+u^2=\frac{c^2}{2h^2(1-2\mu u)^2}+\text{const.} \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}22)$$

whence it appears that the complete integral will involve elliptic functions.

Since the radius vector of the orbit of the planet Mercury is of the order of $5{\cdot}77\times10^7$ *Km*, the magnitude of the term $2\mu u$ is about $0{\cdot}5\times10^{-7}$, and in expanding $(1-2\mu u)^{-3}$ by the binomial theorem to obtain an approximate answer we may neglect higher powers of $2\mu u$ than the first. We thus obtain the equation

$$\frac{d^2u}{d\theta^2}+u=\frac{\mu c^2}{h^2}\,.\,(1+6\mu u)$$

or

$$\frac{d^2u}{d\theta^2}+\kappa^2u=\frac{\mu c^2}{h^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}3)$$

where

$$\kappa^2=1-6\frac{\mu^2c^2}{h^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}31)$$

Hence the equation to the orbit is

$$l\,.\,u=1+\varepsilon\cos\kappa\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}4)$$

where

$$\kappa^2h^2=l\,.\,\mu\,.\,c^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}41)$$

This represents an ellipse, of eccentricity ε and semi-latus-rectum l, the major axis of which slowly revolves about a focus. If a is the semi-axis-major of the orbit,

$$l=a(1-\varepsilon^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3{\cdot}5)$$

Hence the advance of the perihelion in radians per revolution is

$$\frac{2\pi}{\kappa}-2\pi=\frac{6\pi^2\mu c^2}{h^2}$$

$$=\frac{6\pi\gamma M}{c^2a(1-\varepsilon^2)} \quad \text{From (3·41) and (3·5)} \quad . \quad . \quad . \quad (3{\cdot}6)$$

the same result as that obtained on the theories of Einstein and Whitehead—namely, 42·9 seconds of arc per century.

§4. The Deviation of Light-Rays by the Solar Field.

It may be recalled that on relativistic theories the action of a gravitational field on the propagation of light is introduced in the following way. The paths of material particles are obtained as the geodesics of a manifold whose line element has the form

$$ds^2=\sum_1^4{}^{(\mu\nu)}g_{\mu\nu}dx^\mu dx^\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4{\cdot}1)$$

and it is then assumed that the tracks of rays of light may be obtained as the minimal lines of the same manifold. In order to exhibit the effect of a gravitational field according to the theory now being developed, I adopt an hypothesis which seems no more extravagant than that just mentioned.

It appears from equations (2·8) and (2·81) that the paths of material particles are the geodesics of the manifold whose line element is

$$ds^2=(W-V)(dx^2+dy^2+dz^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4{\cdot}2)$$

where W is a constant equal to $\frac{1}{2}\left\{\left(\frac{dx}{d\tau}\right)^2+\left(\frac{dy}{d\tau}\right)^2+\left(\frac{dz}{d\tau}\right)^2\right\}+V$, and determined by the circumstances of the projection of the particle. (ds is, of course, proportional to the element of action of the particle.)

By writing $W=0$ we obtain a form which is particularly simple, and free from any reference to the velocity of projection of a material particle. In this case we find that

$$ds^2=\frac{c^2}{4}\left(1+\frac{2U}{c^2}\right)^{-2}(dx^2+dy^2+dz^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4{\cdot}3)$$

We shall assume that the equations of the electromagnetic field are covariant for all transformations which leave ds^2 invariant. These equations could easily be formed as suggested by Ricci and Levi—Civita [2]; but it is sufficient for our purpose to note that Fermat's principle can now be stated in the form that the paths of light rays render the integral

$$\int\frac{\sqrt{dx^2+dy^2+dz^2}}{c\left(1+\frac{2U}{c^2}\right)}$$

a minimum, the velocity of light being

$$c\left(1+\frac{2U}{c^2}\right)=c(1-2\mu\,.\,u)$$

for the gravitational field of the sun.

This yields, as in the theory of relativity, the result $\frac{4\mu}{R}$ for the deviation in radians of a ray of light passing near the sun, the minimum distance of the ray from the centre of the sun being R.

§5. The Displacement of the Fraunhöfer Lines in the Solar Spectrum.

If we endeavour to imitate the naïve procedure of those relativists who assume that atomic clocks beat out equal values of the ds of equation (4·1) wherever situated, we are led to assume that two similar atoms placed at different points register equal intervals of the proper time τ, so that their true periods of vibration ν_1 and ν_2 are in in the ratio

$$\frac{\nu_1}{\nu_2}=\frac{1-2\mu\,.\,u_2}{1-2\mu\,.\,u_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5{\cdot}1)$$

when situated in the solar field at distances u_1 and u_2 from the sun. This leads to a shift of the solar lines towards the infra-red of an amount

$$\triangle\nu=\left(\frac{2\mu}{R}\right).\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5{\cdot}2)$$

R being the radius of the sun.

This is twice the amount of the Einstein-shift, and although the experimental evidence is at present inconclusive, the existence of the shift given above seems less likely than the existence of the Einstein-shift.

§6. The Variation of Electronic Momentum with Velocity.

In forming the equations of motion of an electron, the only modification which we shall make of classical electrodynamics will be that the mass of the electron varies with its electrostatic energy as described by equation (2·1). Let m and e denote the electronic mass and charge (in electrostatic c.g.s. units), and let E_x, E_y, E_z, H_x, H_y, H_z denote the components of the electric and magnetic intensities in electrostatic and electromagnetic units respectively. Then the equations of motion are

$$\frac{d}{dt}\left(m\frac{dx}{dt}\right)=e\left\{E_x+\frac{1}{c}\left(\frac{dy}{dt}\,.\,H_z-\frac{dz}{dt}H_y\right)\right\}, \text{ \&c.} \quad . \quad . \quad . \quad . \quad . \quad (6{\cdot}1)$$

As in paragraph 2, we transform these equations by taking as our new independent variable the " proper time " τ, defined by the integral

$$\tau=\int_0\frac{m_0\,.\,dt}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6{\cdot}2)$$

taken along the path of the electron from the position occupied at $t=0$. We then find equations (6·1) become

$$m_0\frac{d^2x}{d\tau^2}=e\left\{\frac{m}{m_0}\,.\,E_x+\frac{1}{c}\left(\frac{dy}{d\tau}\,.\,H_z-\frac{dz}{d\tau}H_y\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad (6{\cdot}3)$$

From these equations we deduce the energy-integral

$$\frac{m_0}{2}\left\{\left(\frac{dx}{d\tau}\right)^2+\left(\frac{dy}{d\tau}\right)^2+\left(\frac{dz}{d\tau}\right)^2\right\}=\frac{e}{m_0}\int m(E_x\,.\,dx+E_ydy+E_zdz)+W. \quad . \quad . \quad (6{\cdot}31)$$

where W is a constant equal to the total energy.

Let ψ denote the electrostatic potential, so that

$$E_x=-\frac{\partial\psi}{\partial x},\quad E_y=-\frac{\partial\psi}{\partial y},\quad E_z=-\frac{\partial\psi}{\partial z} \qquad \ldots\ldots\ldots \quad (6{\cdot}4)$$

while the electrostatic energy of the electron is

$$K=e\psi \qquad \ldots\ldots\ldots\ldots \quad (6{\cdot}41)$$

Then from equation (2·1)

$$m=m_0-\frac{e\psi}{c^2} \qquad \ldots\ldots\ldots\ldots \quad (6{\cdot}42)$$

provided that m_0 denotes the mass of the electron in a position where ψ vanishes. Since equations (6·4) and (6·42) yield the result

$$cmE_x=\frac{1}{2}c^2\frac{\partial}{\partial x}\ (m^2),\ \&c. \qquad \ldots\ldots\ldots \quad (6{\cdot}5)$$

we obtain from this last equation and from equation (6·31) the relation

$$\frac{1}{2}m_0\left\{\left(\frac{dx}{d\tau}\right)^2+\left(\frac{dy}{d\tau}\right)^2+\left(\frac{dz}{d\tau}\right)^2\right\}=\frac{c^2m^2}{2m_0}+W \qquad \ldots\ldots\ldots \quad (6{\cdot}51)$$

Let the electron be initially at rest at a point where the electrostatic potential is zero. Then, from equation (6·51) we have that

$$W=-\tfrac{1}{2}m_0c^2 \qquad \ldots\ldots\ldots\ldots \quad (6{\cdot}52)$$

and

$$\frac{1}{2}m_0\left\{\left(\frac{dx}{d\tau}\right)^2+\left(\frac{dy}{d\tau}\right)^2+\left(\frac{dz}{d\tau}\right)^2\right\}=\frac{c^2}{2m_0}(m^2-m_0^2) \qquad \ldots\ldots\ldots \quad (6{\cdot}53)$$

Multiply the last equation by $\left(\frac{d\tau}{dt}\right)^2$, which by equation (6·2) equals $\left(\frac{m_0}{m}\right)^2$, and we obtain

$$\tfrac{1}{2}m_0v^2=\tfrac{1}{2}m_0c^2-\tfrac{1}{2}m_0{}^3.\ m^{-2}c^2$$

where v denotes the resultant velocity of the electron.

Rewriting the last equation, we obtain the familiar result

$$m=\frac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}} \qquad \ldots\ldots\ldots\ldots \quad (6{\cdot}54)$$

which agrees with the Lorentz' formula for the variation of the momentum of an electron with its velocity.

Hence the theory developed in this Paper will interpret the experiments of Kaufman and Bucherer, and similar investigations which study the trajectories of electrons in electromagnetic fields, and which are usually explained by means of the Special Theory of Relativity.

§7. The Fine Structure of the Spectrum of He^+ and H.

There is another class of phenomena, which depend for their elucidation on the variability of electronic mass, but which are not completely covered by the researches of the last paragraph. These are the phenomena of the fine structure of the spectra

of hydrogen and of ionised helium. In the special theory of relativity the kinetic energy of an electron moving with velocity v is taken as

$$m_0c^2\left\{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right\}$$

while the left-hand side of equation (6·52) above is equal to

$$\frac{\frac{1}{2}m_0v^2}{1-\frac{v^2}{c^2}}$$

an expression which plays the part of kinetic energy of the electron in our new theory.

Since the Bohr-Sommerfeld theory of the hydrogen spectrum involves the first expression for the energy, it is necessary for us to show in detail that the same predictions may be made on the basis of the present theory.

It is simpler to abandon the special convention of equations (6·52), (6·53) and (6·54) and to utilise equations (6·3) and (6·51). The potential at any point due to a fixed positive nucleus carrying a charge E (electrostatic units) is E/r, where r is the distance of the point considered from the nucelus. Hence, if m_0 denotes the mass of the electron when removed to an infinite distance, equation (6·42) becomes

$$m=m_0-\frac{eE}{c^2r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot1)$$

while equations (6·3) may be written as

$$\frac{d^2x}{d\tau^2}=-\frac{\partial V}{\partial x}, \text{ \&c.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot2)$$

where $$V=-\frac{c^2}{2}\cdot\left\{1-\frac{eE}{m_0c^2r}\right\}^2=-\frac{c^2}{2}+\frac{eE}{m_0r}-\frac{e^2E^2}{2m_0^2c^2r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot21)$$

It follows that the orbit of the electron has the same form as if a particle of unit mass were moving under the influence of a field of force of potential V in Newtonian dynamics. The equation to the orbit is therefore

$$\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot3)$$

where the central acceleration

$$P=\frac{\partial V}{\partial r}=-\frac{eE}{m_0r^2}+\frac{e^2E^2}{m_0^2c^2r^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot31)$$

and

$$r^2\frac{d\theta}{d\tau}=h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot32)$$

Hence equation (7·3) becomes

$$\frac{d^2u}{d\theta^2}+\kappa^2u=-\frac{eE}{m_0h^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot33)$$

where

$$\kappa^2=1-\left(\frac{eE}{m_0ch}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot34)$$

The equation to the orbit is therefore

$$l \cdot u = 1 + \varepsilon \cos \kappa\theta \quad \ldots\ldots\ldots\ldots \quad (7{\cdot}35)$$

where

$$\kappa^2 = -\frac{eE}{m_0 h^2} \cdot l \quad \ldots\ldots\ldots\ldots \quad (7{\cdot}36)$$

This is precisely the same as on Sommerfeld's theory.

The quantum conditions may be written in the form

$$\int p_r dr = n_1 N, \quad \int_0^{2\pi} p_\theta \cdot d\theta = n_2 N \quad \ldots\ldots\ldots\ldots \quad (7{\cdot}41)$$

where n_1 and n_2 are integers and N is Planck's constant,

where the radial momentum $p_r = m_0 \dfrac{dr}{d\tau}$

and the azimuthal momentum $p_\theta = m_0 r^2 \dfrac{d\theta}{d\tau}$ $\Bigg\}$ $\ldots\ldots\ldots$ (7·42)

As in Sommerfeld's Paper [3], we obtain the equations

$$\left.\begin{aligned} n_1 N &= 2\pi p_\theta \\ n_2 N &= 2\pi\kappa p_\theta \left\{ \frac{1}{\sqrt{1-\varepsilon^2}} - 1 \right\} \end{aligned}\right\} \quad \ldots\ldots \quad (7{\cdot}43)$$

whence

$$1 - \varepsilon^2 = \frac{n_1^2 \kappa^2}{(n_1\kappa + n_2)^2} \quad \ldots\ldots\ldots\ldots \quad (7{\cdot}44)$$

On calculating the value of W at the perihelion ($\theta = 0$, $l \cdot u = 1+\varepsilon$) we find that sinie, r is a minimum,

$$\frac{dr}{dt} = 0$$

while

$$r^2\left(\frac{d\theta}{d\tau}\right)^2 = r^2\left(\frac{h}{r^2}\right)^2 = h^2 u^2 \text{ from (7·32)}$$

therefore

$$W = m_0 V + \frac{1}{2} m_0 \left\{ \left(\frac{d\theta}{d\tau}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 \right\}$$

$$= \frac{eE}{2e}(1-\varepsilon^2) - \frac{m_0 c^2}{2}$$

$$= \frac{2\pi^2 m_0 e^2 E^2}{N^2} \cdot (n_1\kappa + n_2)^{-2} - \frac{m_0 c^2}{2} \quad \ldots\ldots\ldots\ldots \quad (7{\cdot}5)$$

from equations (7·36) and (7·44).

Let R denote the Rydberg constant $\dfrac{2\pi^2 m_0 e^2 E^2}{N^3}$, and α the dimensionless number,

$$\left(\frac{2\pi e E}{N c}\right)^2 = \left(\frac{n_1 e E}{m_0 h c}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (7{\cdot}51)$$

and is approximately $5{\cdot}32 \times 10^{-5}$ for hydrogen.

Then $(n_1\kappa+n_2)^{-2}\doteqdot(n_1+n_2)^{-2}\cdot\left\{1+\frac{\alpha}{n_1(n_1+n_2)}\right\}$ from (7·34) and (7·51), neglecting squares of α. Hence

$$W+\frac{m_0c^2}{2}=\frac{RN}{(n_1+n_2)^2}\left\{1+\frac{\alpha}{(n_1+n_2)^2}\cdot\left(1+\frac{n_2}{n_1}\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot52)$$

Sommerfeld's formula is

$$W=\frac{RN}{(n_1+n_2)^2}\left\{1+\frac{\alpha}{(n_1+n_2)^2}\cdot\left(\frac{1}{4}+\frac{n_2}{n_1}\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7\cdot53)$$

By comparing these formulæ we observe that the "relativistic correction for circular orbits" namely,

$$\frac{RN\alpha}{(n_1+n_2)^4}\cdot\frac{1}{4}$$

is increased fourfold in equation (7·52). The variations in the "effective Rydberg constant" for different lines in the Balmer series due to this cause have not yet been measured sufficiently accurately to permit any comparisons with theory.

The fine-structure of the series is governed entirely by the term

$$\frac{RN\alpha}{(n_1+n_2)^4}\cdot\frac{n_2}{n_1}$$

and this appears in both formulæ (7·52) and (7·53). Hence the details of the fine-structure will be identical on both theories.

§8. Conclusion.

We have shown that the simple assumption made in paragraph 2 connecting the mass of a body with its potential energy, when supported by certain subsidiary hypotheses, is sufficient to account for the following phenomena—the secular advance of the perihelion of the planet Mercury, the deviation of light rays by the sun, the variation of electronic mass with velocity, and the fine-structure of the spectra of hydrogen and of ionised helium. From this it appears that the crucial experiments to test relativistic theories have yet to be devised and performed, as the present theory is sufficient to predict all those phenomena which have hitherto been regarded as the special preserve of the theories of relativity.

References.

(1) J. J. Thomson, "Mass, Energy, and Radiation," Phil. Mag., Vol. 39, pp. 679-689 (1920).

(2) G. Ricci and T. Levi-Civita, "Méthodes de calcul différentiel absolu et leurs applications," Math. Ann. Band 54, pp. 196-7 (1900).

(3) A. Sommerfeld, "Zur Quantentheorie der Spekrallinien," Annalen der Physik, Band 51, pp. 1-94 (1916).

DISCUSSION.

Prof. A. S. EDDINGTON (communicated) : I have had the opportunity of reading Dr. Temple's Paper. I have no quarrel with his mathematics, and have often admired the elegance and ingenuity of his methods. But it seems to me that work of this kind can only appeal to someone who has already been led to believe that the relativity theory is fallacious. Otherwise I cannot imagine why anyone should reject the complete physical theory of Einstein, and prefer the method proposed by Dr. Temple (and others) of an arbitrary tinkering of the fundamental equations. It seems to me that Einstein's method, which consists in rejecting certain hypotheses which had crept into current physics without any experimental support, and re-examining our knowledge unbiased by these traditional hypotheses, goes deeper and is more convincing than any proposal to patch up the Newtonian equations by addition of extra terms.

I do not think that the success in explaining " crucial " phenomena is greater than would be expected from the additional constants introduced. It would be interesting if a term added to explain change of mass with velocity were found also to explain the gravitational deflection of light. But it does not. It gives only half the observed deflection ; and the discrepancy has to be cured by arbitrarily doubling the term. The displacement of the Fraunhöfer lines is not explained ; Dr. Temple tackles it by a procedure justified if the relativity theory is accepted, but admittedly unjustified on his own theory, and obtains a result which the experimental evidence (such as it is) does not seem to favour. The fine structure of the hydrogen lines is scarcely an *additional* success ; the discussion of this phenomenon was required in order to remove an apparent difficulty in Dr. Temple's theory which does not arise in Lorentz's or Einstein's theory. I think the one distinct success is in deriving the perihelion of mercury and the deflection of light from the same formulation of the equations ; but even here it seems that we are required to accept a conclusion of the relativity theory (whilst rejecting the arguments that led to it). Surely the discovery that gravitation acts on light-waves compels us to reject the Newtonian picture of gravitation as a force—you cannot deflect *waves* by tugging at them ; Newton's picture of the invisible agent that pulls down the apple does not correspond to an agency which can deviate a train of waves. We have to find (as Einstein has done) a new picture of the gravitational field which shall represent its action on waves as well as on particles ; and it is this new picture which gives us the hint as to the natural formulation of the law of gravitation.

Mr. F. E. SMITH welcomed the Paper as being a good deal easier to follow than most Papers on relativity. He noted that at one point the author obtained a numerical result different from Einstein's, so that it might subsequently become possible to decide between the two theories experimentally.

Dr. D. OWEN : The author has set out to provide a theory more congenial to physicists than Einstein's. But the effort can scarcely be deemed successful for the reason that it appears to amount to little more than a re-presentation of the theory of Einstein. Thus, the fundamental equation (2·1) of the Paper is simply a statement of the identity of energy and mass, which is one of the remarkable simplifications flowing out of the Einstein postulates. Again, in equation (2·7), the author has to introduce a " proper time," which surely will not strike the physicist as more congenial than the notion of a time specification related to space. In dealing with the shift of the Fraunhöfer lines the author refers rather contemptuously to the " naïve procedure " of assuming a certain behaviour of " atomic clocks," and then proceeds to help himself to the same artifice ! The fact that he arrives at a shift differing in value from Einstein's appears to have no significance, for this merely depends on the choice he has made, apparently quite arbitrarily, of the numerical values of the coefficient λ. Will the author explain why he has adopted different values for this coefficient in the cases of gravitational and electrical fields ? In view of the avowed aim to provide something more acceptable to physicists than Einstein, it would be of interest to learn whether the Author can bring back the æther ; if so, many will doubtless heave a sigh of relief, though there are not a few who have now learnt to resign themselves to the alternative conception of space-time. Whilst offering these criticisms, I nevertheless think this able Paper of great value as holding out the possibility of the proposition of the identity of energy and mass affording a centre from which many of the conclusions of the relativity theory may be readily derived.

Mr. J. E. CALTHROP : I do not see how the theory can interpret the experiments of Kaufmann *and* Bucherer. Kaufmann's experiments supported Abraham's formula, but according to Sommerfeld these results were not sufficiently accurate to distinguish between the Abraham and Lorentz expressions. Later work (Schäfer and Neumann, Ann. d. Phys., 45, 529, 1914) has

supported the Lorentz and relativity result. A reference will be found in Sommerfeld's "Atomic Structure."

Dr. J. S. G. THOMAS said that in introducing his Paper the author had referred to D. C. Miller's results, and to Dr. Silberstein's letter in NATURE describing these as an invalidation of Einstein's theory. Miller's results should, however, be accepted with great caution until more was known about their experimental details, for, as Prof. Eddington had pointed out, the differential ether drift which they imply is entirely contradicted by ordinary astronomical experience. Dr. Owen seemed to be right in asserting that the author's theory is identical in principle with Einstein's. In ordinary dynamics one can write down either the equations of motion of a particle or its energy equations, and the difference between the two methods appeared to be analogous to the difference between the author's and Einstein's methods, so far as essential principles were concerned. It was true that the author makes only one modification of Newtonian mechanics, but that modification contains by implication the whole of Einstein's theory. There might be other lines of approach, such as that which has recently been put forward by Fournier, but all these lines are equivalent to one another in principle.

Mr. T. SMITH said that he felt the author's offering to the experimental physicist was not to be despised. He had offered something which could be envisaged, and was of a familiar character—namely, a mode of behaviour of mass and potential energy—whereas Einstein had dealt in terms of action, which is a far more elusive conception.

Mr. F. E. SMITH said that, after hearing the Paper, he felt he could stand relativity a little better than before. It was gratifying to feel that mathematical physicists could not get much further until experimental physicists supplied them with more ammunition.

Capt. C. W. HUME (communicated subsequently): The metaphysical significance of the Paper seems to lie in the fact that it further delivers us from those strange paradoxes concerning the nature of space and time with which Einstein has troubled our peace. To any one who holds an idealist view of the universe a dispute as to the reality of the ether is a dispute about words, for ether and matter are in any case mere intellectual fictions which (like the virtual image in optics) serve to co-ordinate the elements of our experience. But when we are asked by Minkowski's famous dictum to believe that space and time are in some way identical in their nature, that is a very different matter. It has happened before now, as in the case of Carnot's theorem, that the mathematical structure of a theory has had to be separated from the metaphysical content in association with which it was first put forward: it is possible for the former to be true while the latter is false. The author has shown that the mathematical structure of Einstein's theory can be retained in essentials while an entirely different metaphysic, his or another, is associated with it. May I also venture to suggest that Minkowski's dictum about the identity of time and space is based on a fallacy which arises from taking in a literal sense the loose statement that the time-space systems of two relatively moving *observers* will be different from one another. As a matter of fact, the nature of the universe is precisely the same for all observers, even on the principle of relativity; the orientation of the time-space axes is different for different *systems of measurement*, but any observer can choose any system of measurement he pleases. In the loose phraseology commonly employed, each observer is presumed to use that particular system of measurement which in practice it would be most convenient for him to use; but there is no reason in principle why he should not measure time by a flying clock and measure space with high-speed measuring rods, if he chooses. In connexion with the metaphysics of relativity it is worth noting that the time order of events in the experience of any one monad is the same "for all observers"; or, rather, whatever space-time co-ordinates you adopt, the experiences of any monad will occur in the same order, and this order is the only thing in common between clock measurements and our consciousness of time. And, secondly, any statement which contradicts our immediate intuitive judgments must be fallacious, because the statement itself must at best rest ultimately on intuitive judgments which themselves have no greater authority than these. Now if there is any intuitive judgment that is clear and certain it is this, that time and space are two entirely different conceptions, having nothing in common except a remote analogy between order in space and order in time. Hence, if any theory contradicts this fact, either that theory is untrue or it involves an antinomy in the very nature of thought.

Author's reply: The very kind criticism of this Paper which I have received from Prof. Eddington encourages me to point out an aspect of the subject which appears to have escaped notice. I would be the last to deny the cogency of the observations which Prof. Eddington has made, but I have the uneasy feeling that they tell equally against relativistic theories. Thus,

although Einstein's work is spoken of as a " complete physical theory," based on the rejection of certain unverified hypotheses, a perusal of such a work as Prof. Whitehead's " Principle of Relativity," with its four sets of gravitational equations, shows that there is an arbitrary element in the selection of the Einsteinian gravitational equation $G_{\mu\nu}=0$; while the methods adopted by all relativists in reformulating the equations of the electromagnetic field are frankly selected in order to obtain Maxwell's equations as a first approximation. Such a procedure may not unjustly be described as " tinkering with the fundamental equations." Again, arbitrary constants and even, *horresco referens*, arbitrary functions are not entirely unknown in the General Theory of Relativity. Thus Combridge (Phil. Mag., Vol. 45, pp. 726-732) has shown that the direct integration of the Einsteinian equations for a field possessing spherical symmetry leads to the result (for the line-element of the dynamical manifold)—

$$ds^2=c^2\left(1-\frac{a}{f}\right)dt^2-\left(1-\frac{a}{f}\right)^{-1}.\left(\frac{df}{dr}\right)^2dr^2-f^2(d\theta^2+\sin^2\theta\,.\,d\varphi^2),$$

where a is an arbitrary constant, conveniently identified with $2\gamma M/c^2$ (where γ=the constant of gravitation, c=velocity of light in vacuo, and M=mass of central attracting mass), and f is any arbitrary function of r the radius vector (satisfying two boundary conditions), usually conveniently identified with r itself. There is necessarily an element of contingence in the fundamental equations of any dynamical theory, and all that the present theory can claim is a certain simplicity and intelligibility in its formulation, and a certain agreement with experience in its conclusions. In reply to Dr. Owen, I wish to emphasise the fact that although there is a certain similarity in form between the methods of the present Paper and Einstein's theories, there is a radical diversity of principle. Thus, although equation (2·7) introduces a new variable " τ," named, perhaps somewhat unfortunately, " the proper time," I have deliberately refrained from attaching to this variable any physical significance, such as Minkowski would have attached to the variable bearing the same name in his interpretation of the Special Theory of Relativity ; and I have explicitly stated that the one purpose served by the introduction of this variable is the expression of the equations of motion in Newtonian form—a matter of mathematical convenience rather than of metaphysical necessity. Again, the " identification of mass and energy " in Einstein's theory, alluded to by the same speaker, is a loose, if useful, contraction of a deduction from the Special Theory of Relativity, more accurately expressed in the statement that a flux of kinetic energy w with velocity v is accompanied by a momentum $g=w.v/c^2$ (*see* Chapter XIII., " The Principle of Relativity," E. Cunningham, Camb. Univ. Press, 1914). The deduction assumes that the group of transformations admitted by the equations of particle dynamics is isomorphic with the Lorentz transformation. The assumption of the present Paper is distinct from this result of Einstein's, both in origin and in expression, for it does not presuppose or imply the Lorentz transformation, neither does it contain any reference to kinetic energy. Finally, it appears that both Dr. Owen and Dr. Thomas confuse the special and general theories of relativity. The assumption of that particular consequence of the special theory of relativity, to which allusion has just been made, cannot be used to predict the phenomenon of § 3, neither by itself, nor when supported by the supplementary hypotheses of Poincaré (pp. 173-180, loc. cit.), or of Silberstein (Phil. Mag., Vol. 36., pp. 94-128). The advance of the perihelion of mercury has been explained on relativistic principles, only by the general theories of Einstein and Whitehead, the assumptions of which are destructive of the presuppositions of the present Paper regarding the independence of space and time.

The method of §5 was intended as a *reductio ad absurdum* of the method used in Einstein's theory for predicting the solar spectral shift, and like that method involves an additional *ad hoc* assumption for which no theoretical reason is given, and leads to a result not confirmed by experiment. This criticism is based on the work of Whitehead (" The Principle of Relativity," Camb. Univ. Press, 1922, Chaps. VIII. and XIII.-XV.), who points out that, since electromagnetic forces will be affected by the gravitational field, some knowledge of molecular structure is required for an investigation of the spectral shift.

The theory outlined above implies the negation of an æther only in so far as the existence of an ether is incompatible with the Quantum Theory used in §7. The necessity for using different values in λ in gravitational and electromagnetic problems undoubtedly mars the elegance of the theory. I do not know of any *à priori* justification for these arbitrary values.

In reply to Mr. Calthrop, it appears from pp. 151-3 of Cunningham's textbook quoted above, that although Kaufmann was himself originally of the impression that his experiments supported Abraham's formula, a recalculation of his work, based on a more accurate value of e/m^o, favours the formula of Lorentz.

XXIV.—THE ELECTROSTATIC CAPACITY OF TWO SPHERES WHEN TOUCHING ONE ANOTHER.

By Alexander Russell, *D.Sc., LL.D., F.R.S.*, Principal of Faraday House.

Received May 22, 1925.

Abstract.

Formulæ are given to enable the electrostatic capacity of two unequal spheres in contact to be computed more easily. It is shown that the natural logarithm of any number n can always be expressed as the sum of the capacities of a limited number of pairs of spheres in contact, the number of pairs never being greater than $n/2$. For instance log 7 is the sum of the capacities of the pairs of spheres whose radii are (1, $\frac{1}{6}$), ($\frac{1}{2}$, $\frac{1}{5}$), and ($\frac{1}{3}$, $\frac{1}{4}$) respectively. Several similar relations are pointed out. An exact formula is given for the capacity of the pair of spheres, whose radii are a and b respectively. The formulæ given are a help when computing the capacity coefficients of unequal spheres when very close together.

MATHEMATICAL formulæ for the capacities of two spheres when in contact with one another have been known for many years.* If the radii of the spheres be a and b respectively, so that $a+b$ is the distance between their centres, then, if $C_p\ (a,\ b)$ denote their joint capacity, and if $C_a\ (a,\ b)$ denote the capacity of the sphere whose radius is a when in contact with the sphere whose radius is b, we have

$$C_p(a,\ b)=\frac{ab}{a+b}\left\{-\psi\left(\frac{a}{a+b}\right)-\psi\left(\frac{b}{a+b}\right)-2\gamma\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$C_a(a,\ b)=\frac{1}{2}C_p(a,\ b)+\frac{\pi ab}{2(a+b)}\cot\frac{\pi b}{a+b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$C_b(a,\ b)=\frac{1}{2}C_p(a,\ b)+\frac{\pi ab}{2(a+b)}\cot\frac{\pi a}{a+b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $\psi(x)$ is the logarithmic derivate of the gamma function and γ is Euler's constant (0·577 215 7 . . .). Series formulæ for $\psi(x)$ are given in Todhunter's "Integral Calculus," and a short table of their values is given in Gauss's "Werke," Band 3, pp. 161-162.

As it is now possible to measure the capacity of even a small conductor with high accuracy, the following formula for $C_p\ (a,\ b)$ will be found useful.

When a is not less than b, we have

$$C_p(a,\ b)=a\left\{1+\frac{2b^3}{a(a+b)\ (a+2b)}+0{\cdot}40412\left(\frac{b}{a+b}\right)^3\right.$$
$$\left.+0{\cdot}07386\left(\frac{b}{a+b}\right)^5+0{\cdot}0167\left(\frac{b}{a+b}\right)^7+0{\cdot}004\left(\frac{b}{a+b}\right)^9\right\} \quad . \quad . \quad . \quad . \quad (4)$$

approximately. This formula can be proved at once from the ordinary series formula for $\psi(x)$. Its maximum inaccuracy occurs when a equals b. In this case, putting $a=b=1$, we get

$$C_p\ (1,\ 1)=1{\cdot}386\ 294\ 7\ . \ .$$

The true value$=2\log 2=1{\cdot}386\ 294\ 4\ . \ .$

* A. Russell, Alternating Currents, Vol. 1, p. 243.

As another example, put $a=1$, $b=\frac{1}{3}$, then by (4), (2) and (3) we have

$$C_p\left(1, \frac{1}{3}\right)=1{\cdot}039\ 738\ 8\ \ldots$$

$$C_1\left(1, \frac{1}{3}\right)=\frac{1}{2}C_p\left(1, \frac{1}{3}\right)+\frac{\pi}{8}=0{\cdot}912\ 559\ 5\ \ldots$$

$$C_{\frac{1}{3}}\left(1, \frac{1}{3}\right)=\frac{1}{2}C_p\left(1, \frac{1}{3}\right)-\frac{\pi}{8}=0{\cdot}127\ 179\ 3\ \ldots$$

If we divide these numbers by 900 000 we get the capacities in microfarads. Formulæ (2), (3) and (4) give a practical solution of the problem.

In many cases, however, a simple exact formula can be given for $C_p\,(a, b)$ in terms of logarithmic and trigonometric functions. In proving this formula curious and interesting relations have been found connecting the sums and differences of the electrostatic capacities of pairs of spheres.

If we differentiate Gauss's Multiplication Theorem for gamma functions, we find that

$$\psi(x)+\psi\left(x+\frac{1}{n}\right)+\psi\left(x+\frac{2}{n}\right)+\ \ldots\ +\psi\left(x+\frac{n-1}{n}\right)=n\psi(nx)-n\log n \quad \ldots \quad (5)$$

Putting $x=\frac{1}{n}$ and using (1), we get the following theorems in electrostatics :—

When n is an even integer,

$$C_p\left(1, \frac{1}{n-1}\right)+C_p\left(\frac{1}{2}, \frac{1}{n-2}\right)+\ \ldots\ +C_p\left(\frac{2}{n-2}, \frac{2}{n+2}\right)$$

$$+\frac{1}{2}C_p\left(\frac{2}{n}, \frac{2}{n}\right)=\log_\epsilon n \quad \ldots \quad (6)$$

And when n is odd,

$$C_p\left(1, \frac{1}{n-1}\right)+C_p\left(\frac{1}{2}, \frac{1}{n-2}\right)+\ \ldots\ +C_p\left(\frac{2}{n-1}, \frac{2}{n+1}\right)=\log_\epsilon n \quad \ldots \quad (7)$$

Putting $n=2, 4, 6\ \ldots$ in (6) we get

$$C_p\left(\frac{1}{2}, \frac{1}{2}\right)=\log 2\ ;\ C_p\left(1, \frac{1}{3}\right)+C_p\left(\frac{1}{4}, \frac{1}{4}\right)=\log 4\ ;$$

$$C_p\left(1, \frac{1}{5}\right)+C_p\left(\frac{1}{2}, \frac{1}{4}\right)+C_p\left(\frac{1}{6}, \frac{1}{6}\right)=\log 6\ ;\ \text{etc.}$$

Similarly, putting $n=3, 5, 7, \ldots$ in (7), we have

$$C_p\left(1, \frac{1}{2}\right)=\log 3\ ;\ C_p\left(1, \frac{1}{4}\right)+C_p\left(\frac{1}{2}, \frac{1}{3}\right)=\log 5\ :$$

$$C_p\left(1, \frac{1}{6}\right)+C_p\left(\frac{1}{2}, \frac{1}{5}\right)+C_p\left(\frac{1}{3}, \frac{1}{4}\right)=\log 7\ ;\ \text{etc.}$$

It is curious that the natural logarithm of any number n can always be expressed as the sum of

$$C_p\left(1, \frac{1}{n-1}\right), C_p\left(\frac{1}{2}, \frac{1}{n-2}\right), \ldots$$

the number of pairs of spheres never being greater than $n/2$. Similarly, by writing $1/(2n)$ for x in (5), we get, when n is even,

$$C_p\left(1, \frac{1}{2n-1}\right)+C_p\left(\frac{1}{3}, \frac{1}{2n-3}\right)+ \ldots +C_p\left(\frac{1}{n-1}, \frac{1}{n+1}\right)=\frac{1}{2}\log 4n \quad \ldots \quad (8)$$

And when n is odd,

$$C_p\left(1, \frac{1}{2n-1}\right)+C_p\left(\frac{1}{3}, \frac{1}{2n-3}\right)+ \ldots +\frac{1}{2}C_p\left(\frac{1}{n}, \frac{1}{n}\right)=\frac{1}{2}\log 4n \quad \ldots \quad (9)$$

Many similar theorems can be deduced from the theorems in connexion with $\psi(x)$ given in Bertrand's "Integral Calculus." It suffices to give the following formula:—

$$C_p\left(\frac{1}{n}, \frac{1}{m-n}\right)=\frac{2}{m}\log 2m-\frac{4}{m}\cos\frac{2\pi n}{m}\log\left(\sin\frac{\pi}{m}\right)$$
$$-\frac{4}{m}\cos 2\frac{2\pi n}{m}\log\left(\sin\frac{2\pi}{m}\right)- \ldots -\frac{4}{m}\cos q\frac{2\pi n}{m}\log\left(\sin\frac{q\pi}{m}\right) \quad (10)$$

where q is $\frac{m}{2}-1$ when m is even and $(m-1)/2$ when m is odd. For example, when $m=6$ and $n=1$, (10) simplifies to

$$C_p\left(1, \frac{1}{5}\right)=\frac{1}{2}\log 6+\frac{1}{6}\log 2=1{\cdot}011\ 404\ 3 \ldots$$

To this accuracy we get the same value by (4).

Notice that

$$C_p\left(m, \frac{m}{5}\right)=\frac{1}{2}\log 6^m+\frac{1}{6}\log 2^m.$$

The theorems given above show that there are numerous simple relations connecting the capacities of pairs of spheres which touch one another. For instance, we can easily prove that

$$\log 4=C_p(1,1)=C_p\left(\frac{4}{3}, \frac{4}{9}\right)=C_p\left(1, \frac{1}{7}\right)+C_p\left(\frac{1}{3}, \frac{1}{5}\right)$$
$$=C_p\left(1, \frac{1}{3}\right)+C_p\left(\frac{1}{4}, \frac{1}{4}\right)=C_p\left(2, \frac{2}{11}\right)+C_p\left(\frac{2}{5}, \frac{2}{7}\right)-C_p\left(1, \frac{1}{2}\right)$$
$$=C_p\left(3, \frac{3}{5}\right)-C_p\left(\frac{3}{2}, \frac{3}{4}\right)$$
$$=C_p\left(\frac{2}{1}, \frac{2}{2n-1}\right)-C_p\left(\frac{2}{2}, \frac{2}{2n-2}\right)+C_p\left(\frac{2}{3}, \frac{2}{2n-3}\right)$$
$$- \ldots +(-)^{n+1}C_p\left(\frac{1}{n}, \frac{1}{n}\right),$$

where n is any integer greater than unity.

The following particular cases follow from (10) :—

$$C_p(1\ ,\ 1)=\log 4$$

$$C_p\left(1\ ,\frac{1}{2}\right)=\log 3$$

$$C_p\left(1\ ,\frac{1}{3}\right)=\frac{1}{2}\log 8$$

$$C_p\left(1\ ,\frac{1}{4}\right)=\frac{1}{2}\log 5+\frac{2\sqrt{5}}{10}\log\frac{1+\sqrt{5}}{2}$$

$$C_p\left(1\ ,\frac{1}{5}\right)=\frac{1}{2}\log 6+\frac{1}{6}\log 2$$

$$C_p\left(1\ ,\frac{1}{7}\right)=\log 2+\frac{\sqrt{2}}{4}\log(1+\sqrt{2})$$

$$C_p\left(1\ ,\frac{1}{9}\right)=\frac{1}{5}\log 4+\frac{1}{4}\log 5+\frac{3\sqrt{5}}{10}\log\frac{1+\sqrt{5}}{2}$$

$$C_p\left(1\ ,\frac{1}{11}\right)=\frac{1}{4}\log 12+\frac{\sqrt{3}}{6}\log(2+\sqrt{3}).$$

We also have

$$C_p\left(1\ ,\frac{1}{11}\right)+C_p\left(\frac{1}{5}\ ,\frac{1}{7}\right)=\frac{1}{2}\log 12$$

$$C_p\left(1\ ,\ \frac{1}{7}\right)+C_p\left(\frac{1}{3}\ ,\frac{1}{5}\right)=2\log 2,$$

and

$$C_p\left(1\ ,\ \frac{1}{4}\right)+C_p\left(\frac{1}{2}\ ,\frac{1}{3}\right)=\log 5.$$

We thus find $C_p\left(\frac{1}{5}\ ,\frac{1}{7}\right)$, $C_p\left(\frac{1}{3}\ ,\frac{1}{5}\right)$ and $C_p\left(\frac{1}{2}\ ,\frac{1}{3}\right)$.

A knowledge of the values of $C_p(a,\ b)$ and $C_a\,(a,\ b)$ is a help in computing the capacity coefficients k_{11}, k_{22} and k_{12} of the two spheres when very close together. In this case we have

$$k_{11}+k_{22}+2\ k_{12}=C_p(a,\ b)$$

$$k_{11}+k_{12}=C_a(a,\ b)$$

very approximately, and k_{12} can be found from the known formula for two equal spheres, by means of the formula*

$$k_{12}\ (a,\ b,\ c)=k_{12}(a_1,\ a_1,\ c_1) \qquad \ldots\ldots\ldots\ (11)$$

where

$$a_1{}^2=ab\left\{1-\frac{(a-b)^2}{c^2}\right\}\ \text{and}\ c_1=c\left\{1-\frac{(a-b)^2}{c^2}\right\}.$$

DISCUSSION.

Dr. RUSSELL, in introducing the Paper : I think some explanation is necessary for bringing a mathematical theorem to the notice of the Society. I have found that the computation of the maximum stress in the dielectric between unequal spherical electrodes when electrified, and also of the attraction between them, is very laborious. A knowledge of the values of these quantities is of great importance to the physicist. For instance, with equal spherical electrodes in air the maximum electric stress at the moment of disruptive discharge is a constant at a given temperature

* Proc. Roy. Soc., Vol. 97, p. 166.

and pressure. I would instance the sparking voltages between large spheres which are published annually by the American Institute of Electrical Engineers. In the experiments where the potentials are equal and opposite at the moment of discharge I have found that at 25°C. and 76 cm. pressure the disruptive electric stress is given by $27{\cdot}4+14{\cdot}1/\sqrt{r}$ kv. per cm., where r is the radius of either electrode in cm. I have also found that this expression enables the experimental results found by Heydweiller in 1893 for the sparking voltages between very small spherical electrodes to be accurately computed. In general, for a fixed spark-gap there are certain sized spherical electrodes which offer the maximum resistance to sparking. For example, when the air-gap is 1 cm. the sparking voltage is a maximum for spherical electrodes whose diameters are very nearly equal to two inches. Increasing or diminishing the size of the electrodes from this value makes the sparks take place more readily. It is of interest both practically and theoretically to analyse the sparking voltages between unequal electrodes. A necessary preliminary is to find their capacity coefficients, and as an aid to calculating their values this theorem is published.

Dr. E. H. Rayner: I had an opportunity recently of visiting the transformer works of the General Electric Company of America at Pittsfield, Mass., where much of the work has been done by Mr. Peek and others which has resulted in the adoption of the American figures. As far as I am aware, the figures are derived from an empirical formula made to fit a few points of an experimental character. The accurate measurement of voltages of the order of a few hundred kilovolts, both peak voltages and effective voltages, has, I think, hardly reached a stage at which definite information has been obtained as to the accuracy with which a formula, empirical or otherwise, actually corresponds with the physical facts. The statement of Dr. Russell that a discharge might take the more difficult of two parallel spark gaps is an illustration of the effect of the physical conditions on the relations involved. This actual experiment was shown at Pittsfield, in which two spark gaps two or three inches long were arranged in parallel. The electrodes of one were spheres about four inches in diameter, and the other was a horn type lightning arrester made of rods about half an inch in diameter. At ordinary power frequencies the gaps can be adjusted so that the discharge takes place at the horn type of gap, while at high frequency or impulsive discharges it will change over to the other gap. The reason for this seems to be that at power frequencies there is time for corona to form at the horn gap and ionise the air, and so assist the discharge, while at high frequencies there is not sufficient time for this to be fully effective, and the discharge takes the other path.

Author's reply: The experiment with parallel spark gaps was shown me by Mr. Duddell many years ago. The explanation was fairly obvious. I am not prepared to say that the formula for the disruptive stress is an empirical formula: it is necessary first of all to compute the maximum electric stress, and this involves a large amount of theory. Experiments carried out in many different laboratories during the last forty years have proved that with certain limitations the disruptive stress is a constant for a given pair of equal spherical electrodes. This physical fact was anticipated by the early physicists: when, however, it was found that the disruptive stress varied with the size of the spheres it became desirable to study the problem more closely. From considerations on the convective discharge of electricity I was led to the conclusion that an expression of the form $A+B/\sqrt{r}$, where A and B are constants, would give the disruptive stress, and Whitehead's experiments on the disruptive voltage between parallel cylinders were the first to give the theory strong experimental support. So strong, in fact, is the experimental evidence that I have not the slightest hesitation in applying it in practice, provided that there are no brush discharges in the neighbourhood of the spark gap.

XXV.—CONDENSATION NUCLEI PRODUCED BY THE ILLUMINATION OF AIR-HALOGEN MIXTURES.

By I. C. Jones, *B.Sc.*, Garrod Thomas Research Student, University College, Aberystwyth.

Received March 18, 1925.

Abstract.

It is shown that the production of condensation nuclei, previously known to take place on illumination of air saturated with iodine vapour, also takes place under similar conditions with bromine and chlorine; the usual tests indicate that the nuclei in all three cases are similar.

The effect occurs in two distinct stages:

(1) A surface action dependent on the presence on the surface of traces of some oxidisable substance.

(2) Formation of nuclei in the "activated" vapour by illumination.

A parallel is shown to exist between the conditions under which nuclei are found, and those under which an induction period is observed in the hydrogen-chlorine reaction.

Introductory.

THE present investigation is a continuation of work on the same subject by G. Owen and H. Pealing,* and by H. Pealing.† Much of their work has been repeated and in general confirmed, but a more detailed and definite explanation of the results has been obtained.

The problem arose in the following way: It had been found by G. Owen and A. Ll. Hughes‡ that CO_2 snow, previously condensed in a dust-free state, gave rise on vaporising to immense numbers of particles capable of acting as centres of condensation for water vapour. Similar tests with other subliming substances gave in general negative results, but in the case of iodine a large effect was observed. Later, however, it was shown that in this case the nuclei were not formed if the apparatus was kept in darkness, so that the process producing them was not mechanical, but photochemical.

The apparatus used in the present experiments was the well-known expansion apparatus due to C. T. R. Wilson,§ with which different forms of cloud-chamber could be used as convenience and necessity indicated.

The results produced in saturated dust-free air by expansions of various amounts are so well known that they need not be described in detail; in the present Paper they will be referred to as the "Wilson effects." If the initial and final volumes be respectively v_1 and v_2, B the atmospheric pressure, π the pressure of water vapour

* G. Owen and H. Pealing, Phil. Mag., p. 465, April (1911).
† H. Pealing, Phil. Mag., p. 455 (1915).
‡ G. Owen and A. Ll. Hughes, Phil. Mag., October (1907); June (1908).
§ C. T. R. Wilson, Proc. Camb. Phil. Soc., 9, p. 333 (1897).

at the prevailing temperature, and p_1 and p_2 the initial and final readings of a manometer attached to the expansion chamber, then

$$\frac{v_2}{v_1}=\frac{B-p_1-\pi}{B-p_2-\pi}$$

It is usually convenient to denote the measure of the expansion by the pressure-fall p_2-p_1 produced by it; the calculation of the expansion ratio v_2/v_1 is only occasionally necessary.

The Photochemical Effect.

For the observing of the effect, the type of cloud-chamber shown in Fig. 1 was found convenient. A few crystals of iodine were placed in the cup A, and, after freeing the air from dust, expansions were made, with the results shown below. The expansions "In Light" were made with the apparatus illuminated by a 100-watt lamp about five feet away; to view the clouds, this light was switched off and

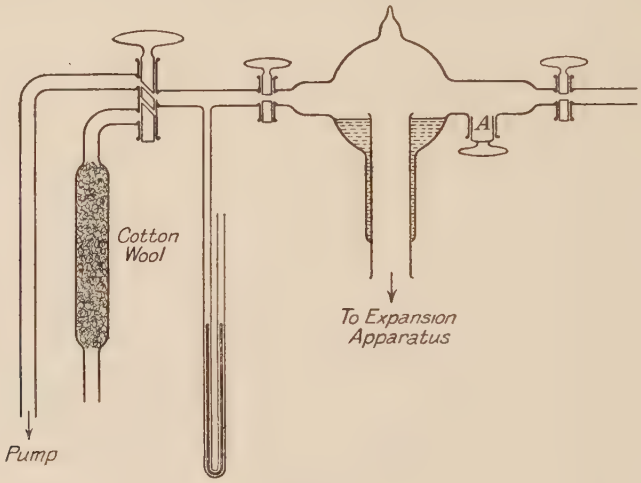

Fig. 1.

a second light put on, so arranged as to send a narrow beam of light through the cloud-chamber, the effects being observed at a small angle to this beam, against a black background. The results of expansions made with the apparatus in darkness are added for comparison.

Pressure-fall (from atmospheric).	*Result of expansion.* *In light.*	*In darkness (Wilson effects).*
15 cm.	Fair shower.	Few drops.
17 cm.	Coarse-grain cloud.	Medium shower.
18·5 cm.	Dense coloured cloud.	Good shower.
20 cm.	Fog.	Fog.

A greatly increased density of cloud is thus observed at 18·5 cm., and also somewhat more nuclei at smaller pressure-falls. This confirms the result obtained by Owen and Pealing.

The Effect in Bromine and Chlorine.

To test whether the nuclei are peculiar to iodine or common to all the halogens, experiments were made with bromine and chlorine. Bromine was introduced by placing a few drops in the cup *A* ; chlorine was passed in from a cylinder. In order to make a fair comparison, the chlorine was greatly diluted with air ; this approximates to the conditions under which bromine and iodine were examined, and also does not greatly alter the value of Cp/Cv. (Cp/Cv being different for chlorine and air, the same pressure-fall will not produce the same cooling, and therefore the same Wilson effect, in both cases.)

The results are given below ; in both cases the nucleation in darkness was the usual Wilson effect.

	Result of expansion (in light).	
Pressure-fall.	*In Bromine.*	*In Chlorine.*
15·0 cm.	Small shower.	—
16·5 cm.	Coarse cloud.	Coarse cloud.
18·0 cm.	Fine cloud.	—
18·5 cm.	Dense coloured cloud.	Dense coloured cloud.
19·0 cm.	Dense coloured cloud.	Dense coloured cloud.

The same effect is seen to be present in both cases as in iodine. In presence of an excess of chlorine, the difference between the " light " and " darkness " effects was not so great as when the chlorine was greatly diluted with air.

Growth and Decay of Nuclei.

In the case of iodine, it was observed by Owen and Pealing that the nuclei take about one second to " grow " in light—i.e., the full density of cloud at 18·5 cm. is not obtained till the vapour has been illuminated for that time. Similarly, the nuclei take about thirty seconds to disappear after the light has been cut off.

Measurements of the rate of growth and decay were made in bromine and chlorine, and found to be the same as those for iodine, as the following tables show :

Rate of Growth.—Pressure-fall throughout, 18·5 cm.

	Result of expansion.	
Time of illumination.	*In Bromine.*	*In Chlorine.*
About $\frac{1}{4}$ sec.	Heavy shower.	Good shower.
$\frac{3}{4}$ sec.	Fine cloud.	Coarse cloud.
1 sec.	Dense coloured cloud.	Fine cloud.
2 sec.	Dense coloured cloud.	Dense coloured cloud.

Rate of Decay.—Pressure-fall throughout, 18·5 cm.

Period of darkness	*Result of expansion.*	
before expansion.	*In Bromine.*	*In Chlorine.*
2 mins.	Shower (Wilson effect).	Shower (Wilson effect).
1 min.	Heavy shower.	Heavy shower.
30 sec.	Very heavy shower.	Very heavy shower.
5 sec.	Dense coloured cloud.	Dense coloured cloud.

It is seen that all the nuclei persist for five seconds, and that some are still present after thirty seconds.

It has now been shown that illumination of a mixture of any one of the common

halogens, with air for a period of 1-2 seconds, produces an immense number of nuclei, caught by an expansion of ratio 1·32-1·33 (pressure-fall 18·5-19 cm.), and disappearing entirely if the light is cut off for thirty seconds or longer. The density of the clouds is, as far as can be judged, the same in all three cases. The assumption is therefore justified that the nuclei are similar in all three cases, that they are produced by the same process, and that in all probability the halogen does not enter into their composition.

The diminution of the effect with time.

One point of difference was observed between the halogens, namely, the rate of diminution of the effect with time. Owen and Pealing state that within four or five days of setting up the apparatus the result of an expansion of 18·5 cm. is merely the usual Wilson effect. Their apparatus was kept in bright daylight ; if kept in darkness the effect decayed much more slowly. No such complete decay was observed in the present work, but this was probably due to its not being practicable to obtain more than weak diffuse daylight. A marked diminution in the density of the clouds was produced by illuminating the apparatus for some hours with an electric arc.

The rate of decay observed in iodine was very small, a slight diminution being the most observed, even after several weeks.

In bromine the rate of decay was also small, but greater than in iodine, a diminution being evident in six or seven days.

In the case of chlorine the effect had completely disappeared in three days.

Since the effect appears to be practically identical in all three halogens, further work was done with iodine only, this being more convenient to manipulate than bromine or chlorine, which require for prolonged experiments an apparatus constructed entirely of glass.

The Mechanism of the Effect.

Owen and Pealing produced strong evidence to show that the nuclei were produced by some action taking place at the glass surface ; after the effect had diminished with time, it could be renewed by taking the apparatus down and washing the walls, whilst introducing fresh air only, without washing the walls, sometimes renewed the effect, but did not generally do so.

It was considered desirable in the present experiments to renew the surface in contact with the vapour without changing the air. For this purpose the apparatus shown in Fig. 2 was used so constructed that the cloud-chamber could be filled with water, this being then replaced by iodine-laden air in which the effect had disappeared. The vessel *A* containing this air, with iodine in a small glass cage as shown, was exposed to bright daylight for about two weeks, and the air then transferred to the cloud-chamber as indicated, the water being run out at the base of the expansion apparatus.

On testing this " fatigued " air for nuclei, the full effect was found to be present.

As, according to Owen and Pealing's observations, the nucleation in the exposed vessel should have disappeared entirely, owing to the prolonged exposure to bright daylight, the above result goes to prove that bringing the air in which the effect has decayed into contact with a freshly-washed glass surface renews the effect completely.

The production of the nuclei is, then, undoubtedly a surface action. It is noticed, however, that the clouds characterising the effect are produced throughout the whole cloud-chamber. If the nuclei are produced in any way *by* the surface, they should be produced *at* the surface ; so that there should be a greater density of cloud near the surface than at a distance.

It was readily shown that the nuclei occur only in the illuminated parts of the apparatus, by using a cloud-chamber with two bulbs, one of which could be illuminated whilst the other was kept in darkness. The difference in the effects in the two bulbs was very striking, the "dark" bulb being almost optically empty in comparison with the " light " bulb, which was filled with a dense white cloud.

To detect a difference, if any, in the cloud density near the surface and far away, a T-shaped cloud-chamber was constructed, the ends of the horizontal part being

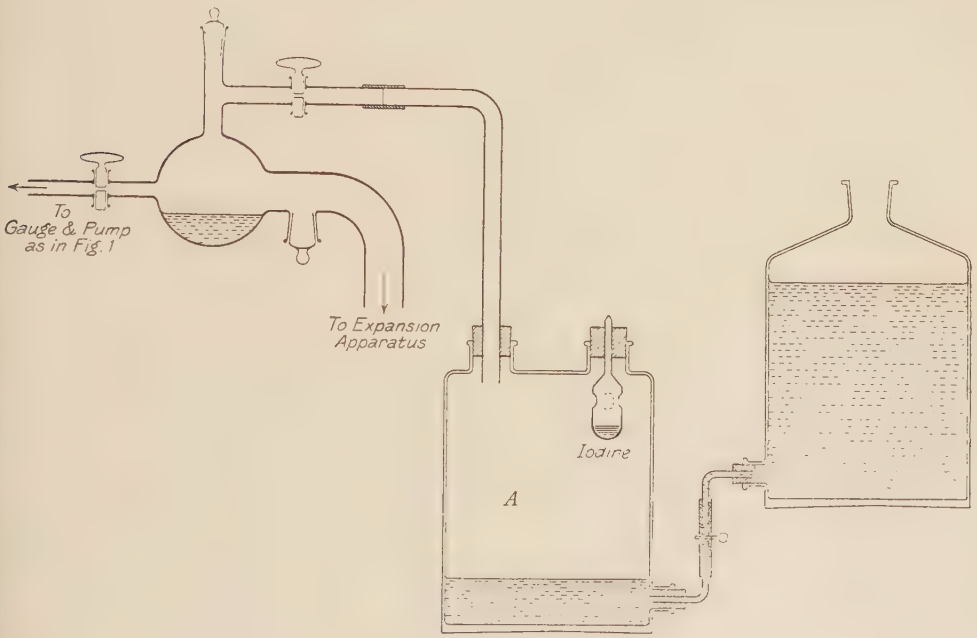

Fig. 2.

ground flat and closed with glass plates. A small beam of light was sent along the axis of this tube, the beam only touching the glass at its entry and exit. On setting, the density of cloud was seen to be the same along the whole path of the beam.

Owing to the peculiar shape of this apparatus, a considerable amount of turbulence occurred on expansion. An apparatus was accordingly designed in which the turbulence was eliminated, but the same result as before was obtained.

It seems therefore that the nuclei are produced, not at the surface, but throughout the whole volume of air illuminated.

Owen and Pealing found that if the iodine vapour was passed into the cloud-chamber along a long plug of glass wool, the effect was much more intense than normally : a more detailed examination of this effect of glass wool has explained

the above apparent anomaly. The experiments were carried out at a time when the "normal" effect in the cloud-chamber was less than usual, so that the differences in the effects could be detected with certainty.

It may be mentioned in passing that under certain conditions it is found that the glass wool gives rise to large nuclei, caught by expansions of the order of 10-12 cm.; but the property of forming these nuclei very soon disappears, and cannot be renewed by cleaning the glass wool—the nuclei are only formed by a freshly-made plug.

With regard to the small nuclei, the following suggestive result was soon observed. While the apparatus was in the non-sensitive state referred to above, iodine vapour and air were drawn through the glass wool into the cloud-chamber, and the light cut off for a few minutes. Making an expansion of 18·5 cm. in the dark, the result was merely the Wilson effect; putting on the light and expanding again, without drawing more air through the glass wool, gave a dense white fog.

This shows that the nuclei are not produced at the glass surface, or even directly by it, but that the mixture of air and iodine vapour undergoes some change in passing through the glass wool which enables it to form nuclei on being illuminated.

An exactly similar circumstance is observed in the case of the large nuclei caught at 10-12 cm.; it follows that the two kinds of nuclei are of the same nature, and differ merely in size.

The effect, then, separates into two parts :—

(1) Some action in the air-iodine mixture at the glass surface;

(2) The formation of nuclei in this "activated" vapour by light. The surface effect has been examined in some detail :—

(*a*) The dense cloud observed with glass wool becomes less dense on repetition—without drawing more air through the glass wool—until after about 20 expansions the original coarse cloud is obtained. This small effect having been restored, the whole apparatus was allowed to stand in darkness for about 30 minutes, and air then drawn through the glass wool, still in darkness. Expanding at 18·5 cm.—good shower; putting on the light and again expanding—dense white cloud.

It is evident that the glass surface which "activates" the vapour need not be illuminated. This shows why the density of cloud throughout the apparatus is the same; the whole body of the gas has already been activated before the light is put on.

(*b*) A fresh plug of glass wool was made up, the glass wool having previously been well dried. The finished plug was also allowed to stand for some days in a desiccator; freshly-sublimed iodine was placed on it, and dry air drawn through it into the cloud-chamber. Expanding at 18·5 cm. in light—dense white cloud.

This result shows that moisture (other than surface films on the glass, which are not removed without heating in a vacuum) is not necessary to the surface effect.

(*c*) Hydrogen was passed through the glass wool and out to the pump for some time, so as to sweep all air from the glass wool and connecting tubes. Then the tap to the cloud-chamber was opened, and the current of hydrogen allowed to pass in for a short time.

Expanding at 18·5 cm. in light—dense white cloud. The presence of oxygen is therefore not necessary to the surface effect.

(*d*) The effect of the glass wool diminished rapidly with time, disappearing entirely in about three days. Attempts were made to renew it by moistening the plug, cleaning it with NaOH and chromic acid, &c., but none of the varied treatments applied had any appreciable effect.

(*e*) It was observed that if the apparatus was allowed to stand in chromic acid for some time before setting up, the effect in it was less than usual ; to see whether the glass wool would be similarly affected a sample of it was allowed to stand for two days in chromic-nitric acid mixture, being afterwards thoroughly washed with water, dried, and made up into a plug. On testing, no appreciable difference could be observed between the clouds produced with and without glass wool.

It appears then, that a thoroughly clean glass surface cannot " activate " the air-iodine mixture.

It was found possible to destroy the " normal " effect in the apparatus itself. Following a suggestion whose origin will appear later, the water to be used in the apparatus was boiled for some time with iodine ; the apparatus was allowed to stand for three days in chromic acid, then thoroughly washed, first with tap-water and finally with the " iodised " water.

On testing for nuclei, the effect was found to be exceedingly small—if it was not being deliberately looked for, it would have passed without notice.

As regards the formation of the nuclei, it was found that a trace only of water was required for their formation ; once the nuclei are formed, any vapour can condense upon them ; for example, in presence of traces of water, the effect is very marked in xylene at pressure-falls of about 16-17 cm.

Conclusion.

It cannot be stated with any degree of certainty what the nuclei are. The evidence goes to show that their production is associated with the presence on the surfaces in contact with the vapour, of some foreign substance which is oxidisable, and which after oxidation ceases to exert any action. The decay of the effect is readily explained by the complete oxidation of this substance ; the oxidation will be more rapid in the case of chlorine than in iodine—causing a more rapid rate of decay. Also the oxidation will proceed more readily in light than in darkness ; with a corresponding effect on the rate of decay, such as is observed. Treatment of the surfaces with chromic acid will oxidise the impurity ; any such substance in the water is removed by boiling with the halogen.

The strange manner in which the effect was observed to vary in intensity from time to time is explained by the presence of a varying amount of impurity in the air of the room ; the presence of foreign matter on the specimens of glass wool used was readily made evident by heating a fresh plug of wool moderately strongly, when the wool became darkened as though carbonised, and gave rise to a faint odour as of burning organic matter. After treatment with chromic acid the glass wool did not behave in this way.

The action of the glass wool is not regarded as being in any way different from that of the glass walls of the apparatus ; the greater intensity of the effect is due to the greater surface and consequently greater amount of foreign adsorbed matter in the case of the glass wool.

There are three possible kinds of surface action which may account for the formation of the nuclei :—

(1) Some action between the iodine and some impurity on the glass.

(2) Some surface action between the glass and iodine or some compound of iodine.

(3) Some catalytic action at the glass surface between iodine, water, and oxygen.

Owen and Pealing considered that the action involved in this case was either (2) or (3), but were unable to reach any definite conclusion. Pealing, in a later Paper, also failed to make any definite statement, though leaning to (3). In this Paper it has been shown fairly conclusively that the surface effect does not take place if the glass is thoroughly clean, so that (1) must represent the correct view.

The view of the effect put forward here coincides to some extent with results obtained by Ramsauer* on the formation of nuclei in air by ultra-violet light. He found that the production of the nuclei was associated with the presence in the air or on the surfaces in contact with it, of small traces of substances like CO_2, NH_3, SO_2, organic vapours, &c. ; when these were carefully removed, by a variety of methods, the nuclei were not formed. He considered that the direct cause of the nucleus formation was an interaction between the ozone generated by the light and these substances. Owen and Pealing suggested that the reactions

$$H_2O+I_2=HIO+HI \quad : \quad HIO+O_2=HI+O_3$$

might be taking place in the present case, but this is doubtful.

It is possible that the product of the surface action may absorb visible radiation and re-emit it as a kind of resonance radiation of such a wavelength as to produce ozone from the air, and accordingly nuclei as in the experiments of Ramsauer.

To summarise, the nuclei appear to be the result of a complex process going on in two distinct stages.

(1) Some action, the nature of which is not known, between the halogen and some oxidisable substance existing under ordinary conditions on all glass surfaces ; this action takes place in darkness and in the absence of water or oxygen.

(2) The vapour, when moistened and mixed with oxygen, after being in contact with the surface, is in a condition to form an immense number of nuclei of a fairly definite and uniform size on being illuminated for a short time. If the amount of surface exposed to the vapour be very large, as in the case of glass wool, the nuclei may grow to an abnormally large size.

Mention may be made of a parallel which seems to exist between the results of this work and those obtained by a number of investigators in connexion with the " activation " of chlorine by light. It is well known that a mixture of hydrogen and chlorine must be exposed to light for some time, varying according to the conditions from seconds to hours, before combination to form HCl takes place. The phenomenon has been very widely examined, but is as yet far from being fully understood. It has been traced by some to an action of the light on the *chlorine* only ; and it is generally found that illuminated chlorine is more chemically active than chlorine kept in darkness.

Weigert† has suggested that when gaseous systems containing chlorine are illuminated, " molecular complexes " are produced, which act catalytically as " reaction nuclei "—in the same way as any heterogeneous catalyst. In support of this he points out that many gaseous reactions which are not themselves sensitive to light can be sensitised by the addition of chlorine, i.e., by providing these nuclear catalysts. The theory is very attractive but is negatived by a considerable body of evidence.

* Ramsauer, Phil. Mag., 23, p. 852 (1912).

† Weigert, Ann. der Phys., 4, pp. 243-266 (1907).

The formation of nuclei in illuminated chlorine has been observed by Aitken,* Bevan,† Burgess and Chapman,‡ Radel§ (by an ultra-microscopic method), and others. In all cases the nuclei are observed only if the chlorine is moist. Bevan attributed them to the presence of complex molecules of an addition compound $Cl_2.H_2O$, or in the presence of hydrogen, $H_2.Cl_2.H_2O$. Later work by Burgess and Chapman showed this explanation to be untenable; they attribute the cloud-formation not to the presence of any unusual nuclei, but merely to the lowering of vapour-pressure of the water by the HCl formed in the reaction

$$2Cl_2+2H_2O=4HCl+O_2$$

The parallel between the facts with regard to chlorine and the present observations on iodine is based on the supposition that the nuclei found in iodine and chlorine are the same; it appears that the conditions under which nuclei are observed in iodine vapour (or chlorine) coincide very closely with those where an induction period is found in the hydrogen-chlorine reaction.

Some points of similarity are tabulated below :—

Nucleation in Iodine.	Induction Period in Chlorine.
1. Water is necessary for the formation of the nuclei.	1. An induction period is observed only when the chlorine is moist.
2. Oxygen is necessary to the nucleus formation.	2. Oxygen is found to act as an "inhibitor," i.e., produces or lengthens the induction period.‖
3. Nuclei are not formed when the glass surfaces involved have been thoroughly cleaned.	3. No induction period is observed in thoroughly clean quartz apparatus.¶
4. Nuclei are not produced if the water used has been previously boiled with iodine, and the apparatus, cleaned as in (3) worked with this water.	4. No induction period is observed if the water is previously heated at 100 with chlorine to destroy volatile nitrogen compounds.**
5. The production of nuclei is prevented by prolonged illumination of the apparatus.	5. Prolonged illumination of the reaction vessel shortens and finally removes the induction period.††
6. If there is originally no nucleation in the apparatus, it increases slightly with time.	6. "Inhibitors" are gradually produced in the reaction vessel on allowing to stand. ‡‡
7. Nuclei are formed in presence of salt solutions.	7. Salt solutions cause an induction period. §§

* Aitken, Proc. Roy. Soc. Edin. (1897).
† Bevan, Phil. Trans., A., p. 347 (1903).
‡ Burgess and Chapman, Journ. Chem. Soc., p. 1423 (1906).
§ Radel, Zeit. Phys. Chem., 95, pp. 378-383 (1920).
‖ Chapman and Macmahon, Journ. Chem. Soc., p. 135 (1909).
¶ Burgess and Chapman, Journ. Chem. Soc., p. 1423 (1906).
** Burgess and Chapman, loc. cit. Baly and Barker, Journ. Chem. Soc., p. 653 (1921).
†† Burgess and Chapman, loc. cit. Wendt, Landauer, and Ewing, J.Am.C.S., p. 2377 (1922).
‡‡ Chapman and Whiston, Journ. Chem. Soc., p. 1264 (1919).
§§ Burgess and Chapman, loc. cit.

It seems that it is the actual nuclei, and not the surface conditions producing them, which are related to the phenomenon of photochemical induction; the suggestion is inevitable that the induction period is caused by the available light energy being used up in the formation of nuclei until a certain state is reached where the light energy is able to energise the chemical reaction.

From this point of view, the suggestion made by Weigert is untenable, since he assumes the nuclei to be essential to the course of the reaction.

The force of the explanation of the cloud-formation given by Burgess and Chapman is in great part removed by the fact that it does not occur in the absence of oxygen; also the part played by the surfaces involved is not accounted for in this way. Furthermore, if the nuclei are produced in the case of chlorine by the reaction previously referred to, then they are presumably produced in bromine and iodine by the same type of reaction. It is well known that of the three reactions

$$2Cl_2 + 2H_2O = 4HCl + O_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$2Br_2 + 2H_2O = 4HBr + O_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$2I_2 + 2H_2O = 4HI + O_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(1) goes almost entirely to the right before attaining equilibrium; (2) does not proceed so far, whilst (3) hardly goes at all in this direction. It is difficult to see how this difference in the reactions can be reconciled with the observation that the nuclei in the present experiments are produced with equal readiness and abundance in all three cases.

Further work may show that this parallel is without foundation; but it is worthy of notice.

The writer wishes to express his very great indebtedness to Professor G. Owen for inspiration, advice, and encouragement during the course of this work.

XXVI.—THE CHARACTERISTIC CURVES OF LIQUID JETS.

By E. Tyler, *M.Sc.*, and E. G. Richardson, *B.A.*, *M.Sc.*, *Ph.D.* (University College, London).

Received May 7, 1925.

Abstract.

Continuing the work of S. W. J. Smith and H. Moss upon the relation between the length of a capillary jet and its velocity of efflux from a cylindrical orifice, further examination has been made of the causes to which the main features of the curves obtained by these authors are due. Such curves consist of two main branches. In the first, with increasing velocity, the jet length rises until a critical point is reached. In the second, which begins at this point, the jet length diminishes rapidly with further increase of velocity.

The results contained in the present Paper indicate that, while surface tension is of prime importance in the first parts of these curves, viscosity is the dominating factor in the second.

A general dimensional formula which covers both cases is derived.

Introductory.

THE object of this research was to investigate further the behaviour of liquids issuing from a capillary tube under varying experimental conditions. Curves showing the relation between the "jet-length" (continuous portion of the cylindrical column of liquid) and "velocity of efflux" had previously been obtained by Prof. S. W. J. Smith and H. Moss,* and it was suggested to one of us (E. T.) to investigate the phenomena further. The work was commenced (Part I) in the Physical Laboratory at the University of Birmingham, under the supervision of Prof. Smith, and was later continued by us (Part II) in the Physical Laboratory at University College, London.

Part I.

The apparatus for producing the jets consisted of a metal cylinder, supported in a vertical position, provided with a water-gauge, and terminating in two axial tubes. To the lower of these the nozzle was attached by a short wired-on rubber tube. The upper outlet of the cylinder led to a reservoir, into which air could be pumped through a valve. The procedure in taking a set of readings was to fill the cylinder with the liquid, attach the nozzle, and gradually increase the pressure, registered on a mercury manometer, step by step by means of the pump. The air reservoir kept the pressure constant on the liquid, while readings of the jet-length L (by cathetometer) and the velocity of efflux V (by the volume of issuing liquid collected in a minute) were taken. The jet was illuminated by light from behind and from one side, the "end of the continuous portion" being taken as that point at which the two vertical bands of light reflected in the jet appeared to break up into diffuse light throughout the section of the jet.

The glass nozzles were constructed by drawing out capillary tubing (1 mm. bore), care being taken that the orifice at the point where the tube was subsequently cut was circular. The wax nozzles mentioned later were made by placing a capillary rod in a glass tube about $\frac{1}{2}$ cm. in diameter, slightly coned at the lower end to prevent

* Proc. Roy. Soc., A. 93, 373 (1917).

the wax being forced out by pressure, the interspace being then filled with molten paraffin-wax. The rod was withdrawn when the latter had set.

Typical "L/V curves" are shown in Figs. 1 and 2. They consist of a short

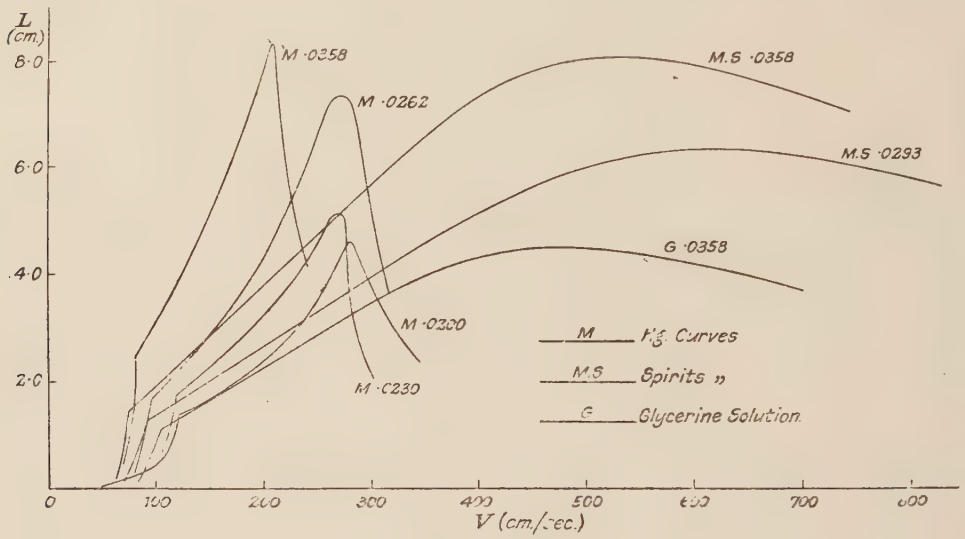

FIG. 1.

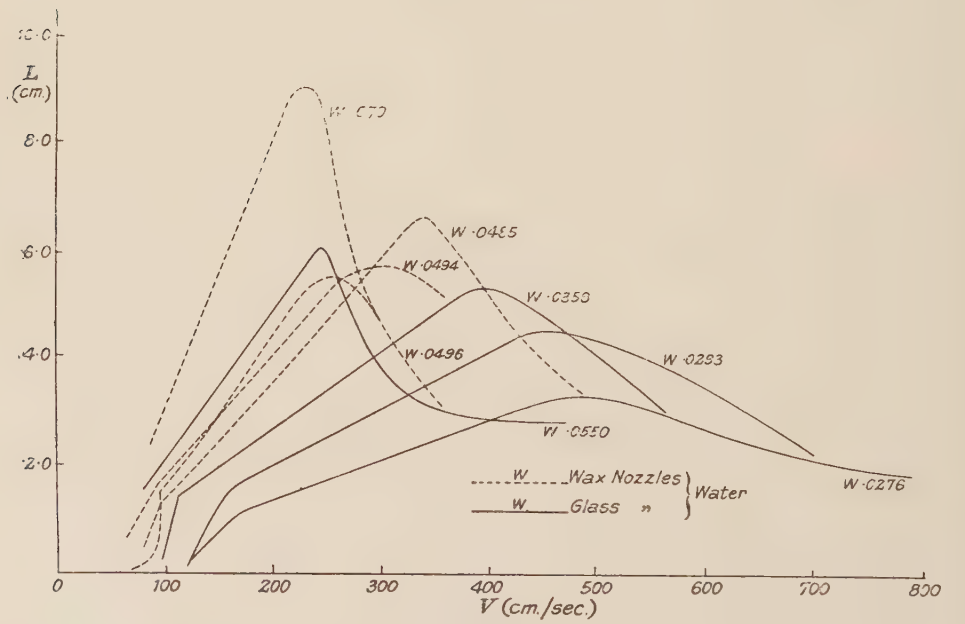

FIG. 2.

curved line, followed by a "kink" (at the "lower critical point"—Smith and Moss); and a straight line at less but still positive slope, until at another stage (the "upper

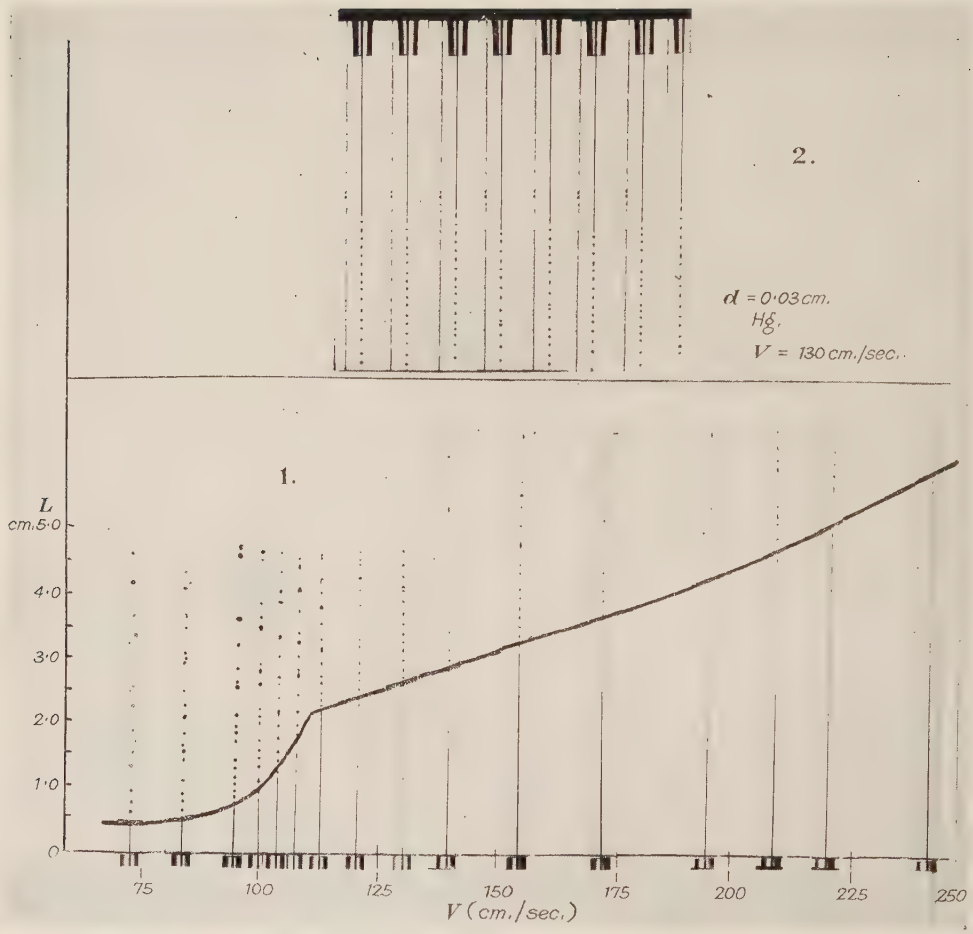

PLATE 1 AND 2.

[*To face page* 299.

critical point"), the slope suddenly becomes negative, L falling in a curve of hyperbolic shape to a nearly constant value, and never actually reaching zero. The two principal parts of the curve—i.e., lower to upper critical points—beyond the upper critical point—will be distinguished as positive and negative parts respectively.

Photographs of the mercury jets were taken in order to check the jet-length as measured by the cathetometer. Instantaneous photographs were taken, using the spark discharge of a Leyden jar as illuminant. Some of the jet photographs, all obtained with the same nozzle (d=0·030 cm., slightly convergent), are shown on Plate 1, placed in appropriate positions on the $L:V$ curve for the same nozzle, which had previously been obtained using the cathetometer to measure L. Having regard to the fluctuations which are always present—cf. Plate 2, which exhibits a series of photographs taken at constant V—it can be seen that the lengths as obtained by the two methods are in substantial agreement. The error in measurement of L is certainly not greater than that involved in V.

Similar curves were obtained by Smith and Moss, who proceeded to develop a theory of Lord Rayleigh, which will be discussed later, and obtained a formula for the positive part of the curve, by which they were able to reduce these positive parts to a single straight line (except in the case of mercury), and to give an interpretation of the linear relationship obtained. If we regard this linear relationship as an experimental fact, the formula (1) below can be deduced from dimensional principles.

Reduction Formula for the Positive Part.—Ignoring for the present the influence of viscosity, L may depend on the diameter of the nozzle (d), the density of the liquid (ρ), its surface tension (σ), and, of course, on V.

Let $$L=\kappa\rho^x d^y\sigma^w V^n$$

Equating exponents of length, mass and time, we get

$$1=-3x+y+n$$
$$0=-2w-n$$
$$0=x+w$$

whence $$L=\kappa\rho^{\frac{n}{2}}d^{1+\frac{n}{2}}\sigma^{-\frac{n}{2}}V^n$$

But experiment shows that L is proportional to V for a given jet; therefore

$$L=\kappa\rho^{\frac{1}{2}}d^{\frac{3}{2}}\sigma^{-\frac{1}{2}}V$$

or $$L/d=\text{constant}\times V\sqrt{\frac{\rho d}{\sigma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The authors cited obtained only a few curves to test this relation, and the curves in Figs. 1 and 2 were obtained in order to test equation (1) further. In Fig. 3, instead of L against V simply, L/d is plotted against $V\sqrt{\frac{\rho d}{\sigma}}$.

Figure 3, page 300, shows that, by means of this transformation, following Smith and Moss, it is possible to represent the positive parts of all the curves by one straight line, which, if produced, would pass through the origin, with the aforementioned exception of mercury. It was thought that this discrepancy might be due to the fact that mercury, unlike water, does not wet glass, and therefore some wax nozzles were made. The results for these nozzles are shown on Fig. 2 by dotted lines, and

with the above transformation on Fig. 3. The fact that the positive parts for wax nozzles lie approximately on the glass nozzle curves show that this explanation—the wetting effect—is not valid.

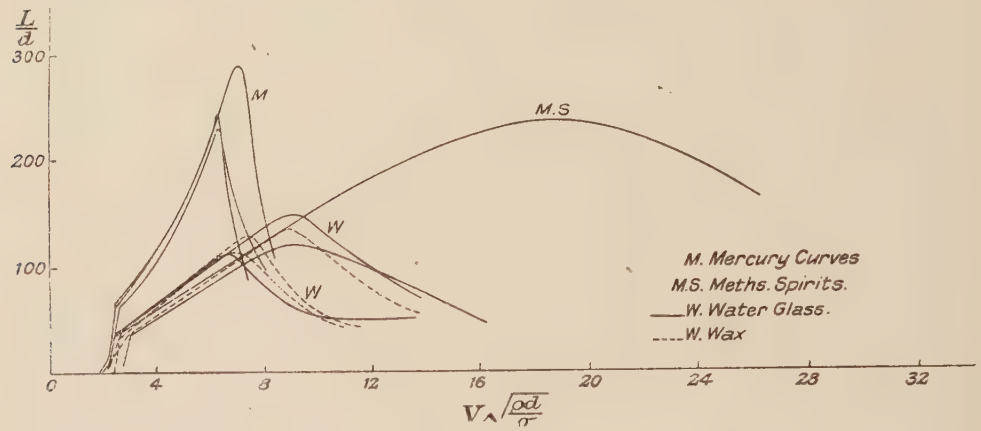

FIG. 3.

The Critical Velocities.—The values of the two critical velocities V_1 and V_2, and also $V_1\sqrt{\frac{\rho d}{\sigma}}$ and $V_2\sqrt{\frac{\rho d}{\sigma}}$, are shown in the following Table A for the liquids water, methylated spirit, and mercury.

TABLE A.

Liquid.	d cm.	Temp. ° C.	σ dynes / cm.	ρ gm. / cm.^{3}	V_1 cm. / sec.	V_2 cm. / sec.	$V_1\sqrt{\frac{\rho d}{\sigma}}$	$V_2\sqrt{\frac{\rho d}{\sigma}}$	$\frac{\mu}{\sqrt{\sigma\rho d}}\times 10^4$
Water (Glass Nozzle)	0·0550	20	72·3	1·00	...	246	...	6·8	50·0
	0·0358	13	74·0	1·00	110	400	2.38	8·7	73·6
	0·0293	15	74·0	1·00	155	455	3·00	8·8	77·3
	0·0276	15	70·5	1·00	160	490	3·16	9·7	81·9
	*0·02325	(17)	74·0	1·00	160	525	2·84	9·3	82·7
Water (Wax Nozzle)	0·070	20	72·3	1·00	...	235	...	7·3	44·2
	0·0496	20	72·3	1·00	95	260	2·50	6·8	52·7
	0·0494	15	74·0	1·00	98	310	2·53	8·0	59·6
	0·0485	16	72·3	1·00	100	340	2·60	8·8	59·8
Mercury (Glass Nozzle)	0·0358	15	547†	13·5	85	210	2·53	6·25	...
	0·0262	15	547	13·5	94	275	2·30	6·87	...
	0·0230	15	547	13·5	117	270	2·81	6·46	...
	0·0200	15	547	13·5	117	282	2·60	6·28	...
	*0·02325	(17)	547	13·5	125	405	3·00	9·70	...
Meth. Spirit (Glass Nozzle)	0·0358	15	24·9	0·83	75	550	2·36	19·0	213
	0·0293	17	24·9	0·83	95	625	2·71	19·5	223
	*0·02325	(13)	24·4	0·82	100	800	2·80	22·4	251

* Smith and Moss values.

† Assumed value from Quincke : Ann. der Phys., 52, 1 (1894).

Comparison with a similar table given by Smith and Moss (loc. cit.) shows that V_1 and V_2 are generally lower than those obtained in their research. This was attributed to the relative convergence of the nozzles used, and a series of experiments were now made, at Prof. Smith's suggestion, to test this point, using mercury and a single glass nozzle of the section shown in the next Figure (4), cut at successive points so as to vary the " taper " of the bore at the orifice.

Analysing these results, one sees that for a perfectly cylindrical nozzle—e.g., d=0·0246 cm.—the value of $V_2\sqrt{\frac{\rho d}{\sigma}}$ is smaller than for one of greater convergency, whereas increase in the divergency (trumpet shape) lowers this value. Hence, the absolute value of this function $V_2\sqrt{\frac{\rho d}{\sigma}}$ will depend on the method of manufacture

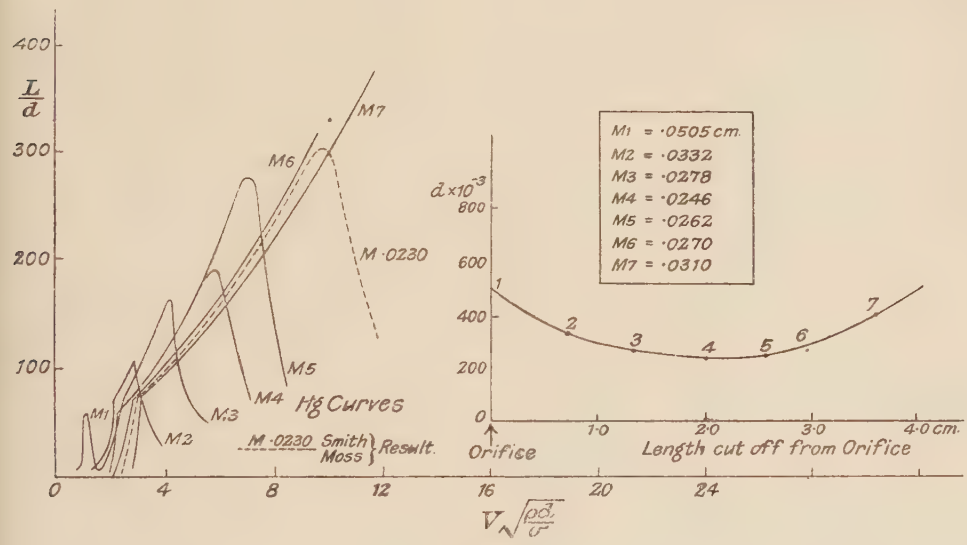

Fig. 4.

of the nozzles, and the discrepancy between these results and those obtained in the earlier Paper is explainable from this point of view.

Effect of Viscosity on Upper Critical Velocity.—Referring to the results for spirit in Fig. 3, it will be seen that the slope of the positive part of the curve is the same as for water, except that the upper critical point is higher ($V_2\sqrt{\frac{\rho d}{\sigma}}$ is about 19 for spirit against 9 for water). The higher value was attributed* to the higher viscosity of methylated spirit, which is 61 per cent. greater than that of water. It was therefore of interest to test the effect of viscosity on the value of $V_2\sqrt{\frac{\rho d}{\sigma}}$ at the upper critical point, and solutions of glycerine and water were employed next (their surface tensions were approximately the same as that of water). One typical $L : V$

* Cf. Smith and Moss, loc. cit., p. 386.

curve is shown in Fig. 1 ($d=0{\cdot}0358$ cm.), the viscosity of the liquid being 94 per cent. greater than that of water. Further results are shown in Table B, with corresponding values of $\frac{\mu}{\sqrt{\sigma\rho d}}$.

TABLE B.—*Viscosity Results* (Water-Glycerine Solutions).

d cm.	Temp. °C.	σ dynes/cm.	ρ gm./cm.^{3}	V_1 cm./sec.	V_2 cm./sec.	$V_1\sqrt{\frac{\rho d}{\sigma}}$	$V_2\sqrt{\frac{\rho d}{\sigma}}$	$\frac{\mu}{\sqrt{\sigma\rho d}}\times 10^4$
0·0358	20	71·2	1·06(4)	105	500	2·5	11·6	117·5
0·0358	20	71·8	1·04(8)	...	452	...	10·3	96·8
0·0358	20	74·3	1·03(5)	...	454	...	10·1	83·2
0·0358	20	68·9	1·02(3)	...	402	...	9·2	80·5
0·0358	18	75·5	1·01(4)	...	410	...	9·0	71·9
0·0358	13	74·0	1·00	...	400	...	8·7	73·6

Suggested Dimensional Theory to account for the Upper Critical Point.—Expressing V in terms of σ, ρ, d and μ, Smith and Moss* have obtained the following dimensional equation for the upper critical velocity,

$$V_2=\kappa\sigma^x\mu^{1-2x}(\rho d)^{x-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where x is an undetermined index, and κ is a constant.

Starting from this equation, we put $1-2x=n$ in (2), and derive

$$V_2=\kappa\sqrt{\frac{\sigma}{\rho d}}\left(\frac{\mu}{\sqrt{\sigma\rho d}}\right)^n$$

In this expression, $\sqrt{\frac{\sigma}{\rho d}}$ is of the dimensions of a velocity, whereas the other term is non-dimensional. Consequently, the general form is

$$V_2=\kappa\sqrt{\frac{\sigma}{\rho d}}\cdot f\left(\frac{\mu}{\sqrt{\sigma\rho d}}\right)$$

If now the function f be expanded as a linear series, the second and higher powers of $\frac{\mu}{\sqrt{\sigma\rho d}}$ may be neglected, since this quantity is small, so that the simplest possible form of the equation, giving the upper critical point, when the effects of viscosity are not negligible, is

$$V_2=\kappa\sqrt{\frac{\sigma}{\rho d}}\left\{a+\frac{b\mu}{\sqrt{\sigma\rho d}}\right\}$$

or

$$V_2\sqrt{\frac{\rho d}{\sigma}}=A_1+B_1\frac{\mu}{\sqrt{\sigma\rho d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where A and B are constants.

Equation (3) is a straight line not passing through the origin. In Fig. 5 the two variables of (3) are plotted, taking the critical values from Tables A and B, and from the various $L:V$ curves in the figures. The graph agrees well with the theoretical forecast. *The upper critical velocity, therefore, involves both surface tension and viscosity.*

* loc. cit., p. 376

PART II.

Viscosity Effect (*con.*).—The object now was to vary viscosity, keeping surface tension constant as far as possible, and therefore some sugar solutions were made of

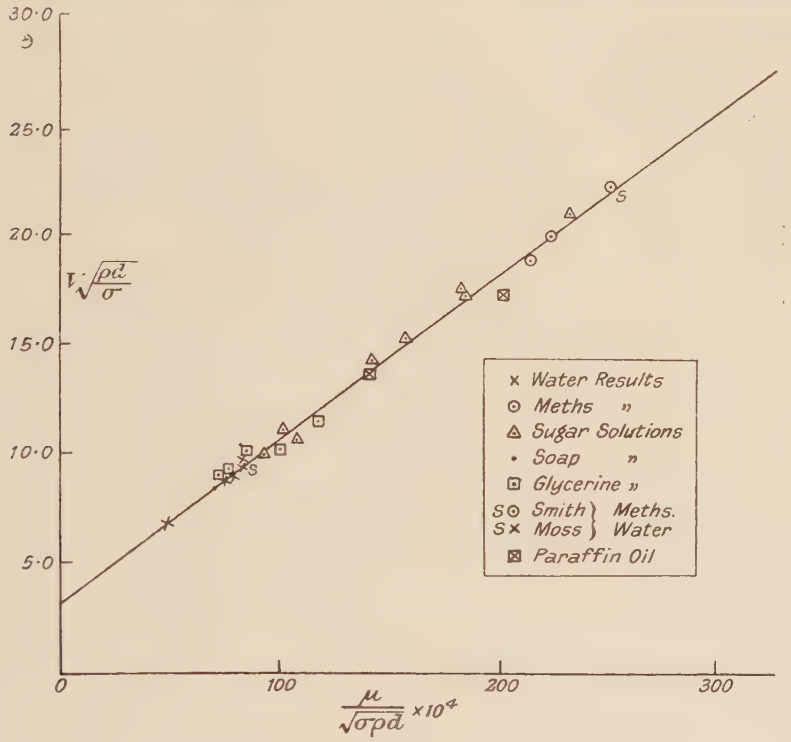

FIG. 5.

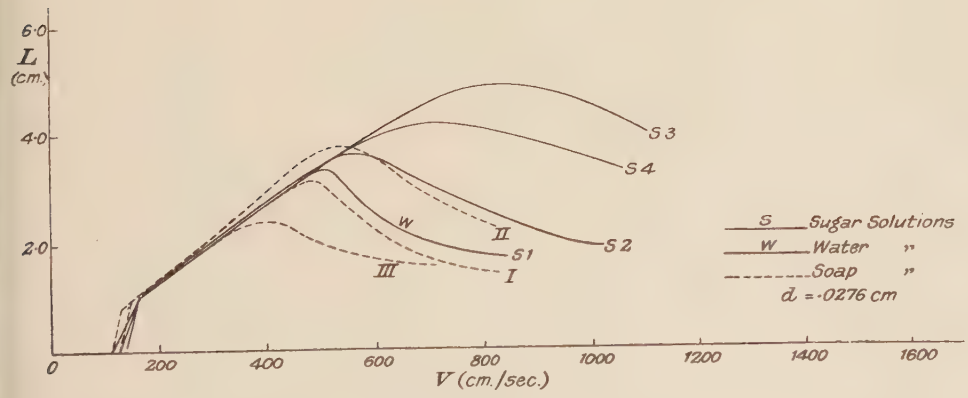

FIG. 6.

various concentrations, as as to produce liquids up to 300 per cent. more viscous than water. Curves for spirit and paraffin oil were also obtained (Figs. 6, 7, 8),

and, with a view to varying the surface tension without the viscosity, soap solutions were tried. But the positive slopes of the soap curves coincided nearly with the

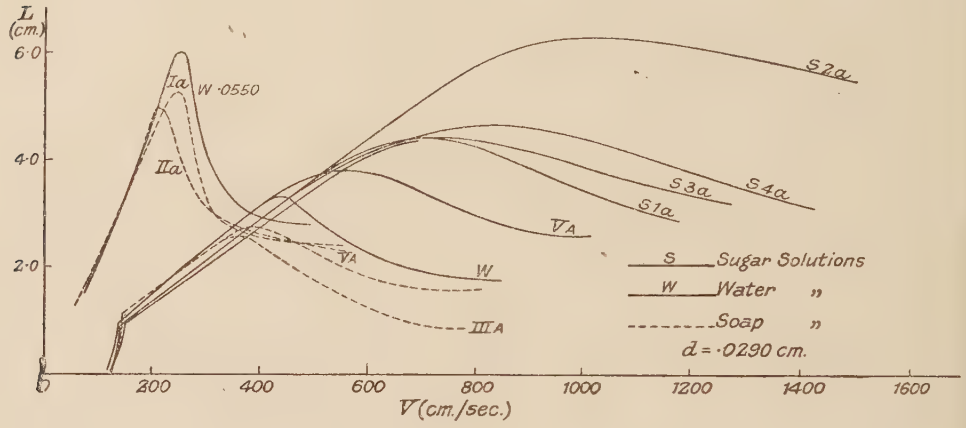

FIG. 7.

water curves, due, no doubt, to the surface tension of the freshly formed soap solution being nearly as high as that of water (vide Lord Rayleigh*).

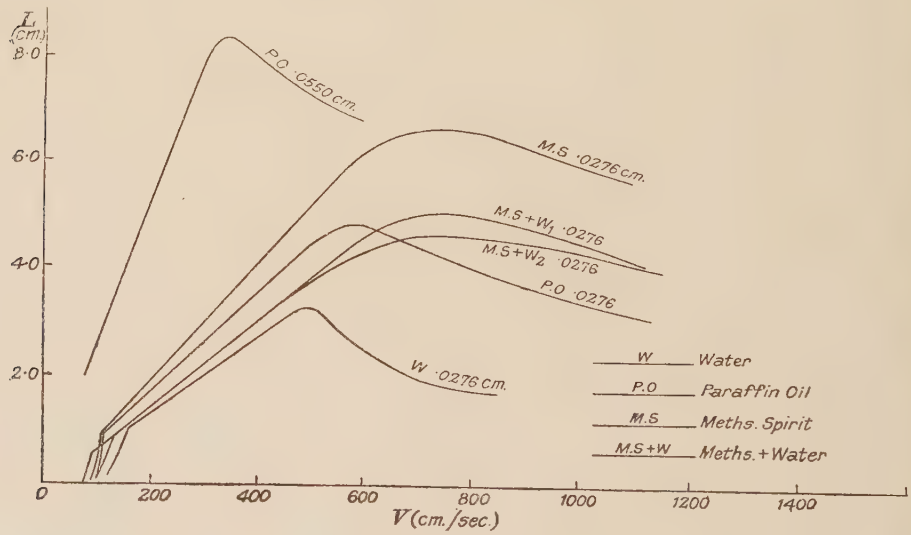

FIG. 8.

In confirmation of previous results, the sugar, soap and water positive parts practically coincide (σ unchanged), the upper critical points (both L and V) are

* Proc. Roy. Soc., 47, 281 (1890).

raised by increasing the viscosity. The reduced curve for the whole series is shown in Fig. 9.

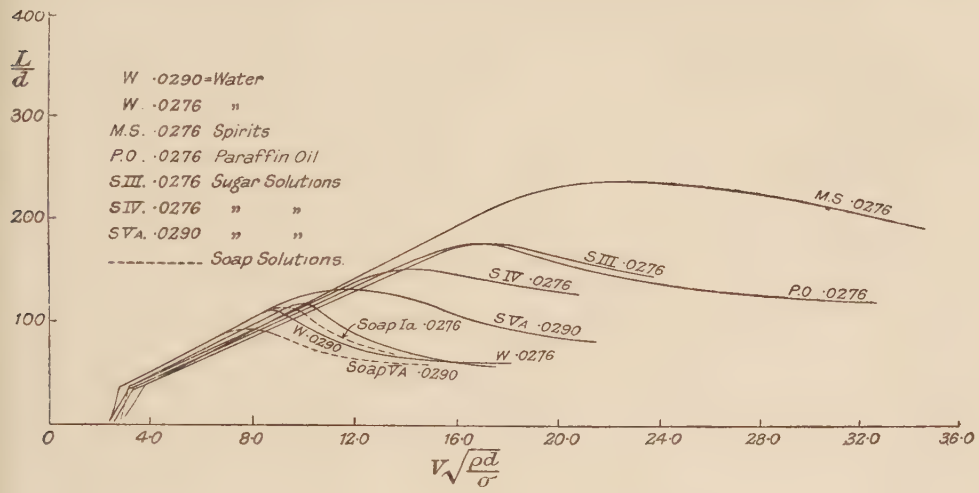

FIG. 9.

TABLE C.

Liquid.	d cm.	Temp. ° C.	σ dynes/cm.	ρ gm./cm.^{3}	V_1 cm./sec.	V_2 cm./sec.	$V_1\sqrt{\frac{\rho d}{\sigma}}$	$V_2\sqrt{\frac{\rho d}{\sigma}}$	$\frac{\mu}{\sqrt{\sigma\rho d}}\times 10^4$
Water.	0·0290	17	72·3	1·00	139	437	2·79	8·77	74·7
	0·0276	15	70·5	1·00	160	490	3·16	9·7	81·9
Paraffin.	0·0550	20	26·4	0·804	...	340	...	13·9	140·5
	0·0276	20	26·4	0·804	110	590	3·19	17·15	199·0
Sugar Solution 1	0·0276	15	71·0	1·017	160	502	3·16	10·0	88·5
Sugar Solution 2	0·0276	18	71·7	1·055	162	549	3·20	10·9	107·8
Sugar Solution 3	0·0276	18	71·8	1·120	160	825	3·32	17·2	180·3
Sugar Solution 4	0·0276	16	72·7	1·085	160	711	3·25	14·4	141·0
Sugar Solution 1a	0·0290	17	73·9	1·090	140	710	2·90	14·70	141·0
Sugar Solution 2a	0·0290	14	73·3	1·135	150	1000	3·18	21·21	232·0
Sugar Solution 3a	0·0290	16	73·3	1·098	140	740	2·91	15·40	155·0
Sugar Solution 4a	0·0290	16	73·0	1·115	130	841	2·72	17·62	178·5
Sugar Solution 5a	0·0290	16	72·3	1·053	140	557	2·86	11·45	101·6
Meth. S.	0·0276	17	23·7	0·813	118	727	3·65	22·47	250·0
Meth. S.-Water.	0·0276	18	53·0	0·95	105	767	2·3	17·15	180·5
	0·0276	15	43·2	0·98	130	753	3·2	18·80	210·0
Soap Soln.	0·0550	19	...	0·99	...	250	...	6·9*	50·0*
	0·0276	15	...	0·99	...	470	...	9·3	81·9
	0·0290	15	...	0·99	140	490	...	8·32	69·8

* Assuming $\sigma=73$ dynes/cm.

These results are included on Fig. 5.

Suggested Theory.

We have now to account for the following characteristics of the curves :—

(1) The positive part.

(2) The negative part.

(3) The upper critical point.

(4) The lower critical point, and the asymptotic tendency of the negative part when V is much increased.

Plateau* first attempted to explain the breaking-up of a cylindrical column of liquid, and showed that when a disturbance acted upon such a column it produced alternate swellings and contractions under the action of surface tension. If the "wave-length" of these appearances (λ) was greater than the circumference of the jet, they grew until finally the jet was split up into separate drops. Rayleigh,† adopting this theory, has shown that, when Plateau's criterion is satisfied, any disturbance impressed upon the liquid column at the orifice will produce waves, which will travel downwards, the amplitude of the disturbance growing exponentially until, finally, it will break up the jet. The rate of growth of this disturbance is most rapid when $\lambda=4{\cdot}51\times d$, and the time of disintegration in this case is given by

$$t=0{\cdot}696\sqrt{\frac{\rho d^3}{\sigma}}$$

Scheuerman,‡ working out the velocity of capillary waves, moving on a liquid jet. has obtained a formula identical with Rayleigh's ; and he supports the latter's theory of the cause of disintegration, as opposed to that of Buff,§ who supposed the swellings to be produced by the formation of drops at the end of the jet, the swellings in turn producing waves that travel *up* the jet. Smith and Moss point out that, with a given velocity of efflux V, the continuous length of the jet L is given by $L=Vt$ or, assuming the maximum instability case, by

$$L=0{\cdot}696V\sqrt{\frac{\rho d^3}{\sigma}}\log\frac{a}{a_0}$$

where a_0=initial amplitude of disturbance at the orifice and a=final amplitude of disturbance causing disintegration ; and that, in this event, the significance of the experimental result

$$\frac{L}{d}=\text{constant}\times V\sqrt{\frac{\rho d}{\sigma}}$$

is (l.c., p. 388) that the ratio a/a_0 is constant.

The Positive Part.—This, following Smith and Moss, we consider to be an effect of surface tension alone—i.e., apart from viscosity.

The Negative Part.—The shape of this part of the curve, and the turbulence which determines the continuous length of the jet, we consider to be due to viscosity, surface tension playing a very small part in producing it. The "end of the jet"

* Statique des Liquides, p. 354 (1873).
† Sound, II, 360.
‡ Ann. der Phys., 60, 234 (1919).
§ Ann. de Chim., 78, 162 (1851).

here represents the point in the accelerating liquid, at which the velocity of adjacent filaments reaches that value, at which, with the given jet, the Osborne Reynolds' criterion for breakdown of the stream-line into turbulent motion—i.e., the stream velocity—has reached a definite value V at this point in the jet where the diameter is D. Increase of velocity of efflux will bring this point nearer the nozzle ; L would be expected to decrease with increasing V, as it does. In support of this idea, the position of the negative parts of the curves on Figs. 7 and 8 may be noted. With a given liquid, decrease of " d " increases all the velocities ; with a given jet, increase of " dynamical viscosity " also moves these parts bodily in the direction of greater V.

In order to test this theory—i.e., that the negative part was a viscosity effect, we thought that if we could remove surface tension from the liquid, we should get the negative part alone, and that L would continually decrease as V was increased. In order to accomplish this, a jet of water coloured with Meldola's blue, which left its density and viscosity unchanged, was allowed to fall from a variable " head " into clear water, thus removing the surface between the jet and the air, although, of course, a discontinuity still existed between the incoming water and the still water in the tank. The results justified expectation ; at small heads, and in the absence of external disturbances, the blue liquid fell in an unbroken column to the bottom of the tank. On increasing V by raising the head, a turbulent point appeared in the jet near the bottom, rose quickly to within 4 cms. of the orifice, and then more gradually to $\frac{1}{2}$ cm. with the greatest heads we could use. The $L : V$ curve for this submerged nozzle was thus hyberbolic in shape (Fig. 10) and lay with fairly good agreement, considering the rather different conditions, on the corresponding $L : V$ (negative) curves for the ordinary water-air jet, in the case of the two diameters (0·0550 and 0·0290 cm.) used. V in this new experiment had to be obtained from the head by the Torricellian formula $V=\kappa\sqrt{2gH}$, which is only approximate for these narrow jets. Furthermore, by taking the ratio of corresponding values of V which produced the same L, we found this ratio to be nearly constant, and the same as the inverse ratio of the respective diameters. That is, V is proportional to $1/d$, which would be expected from a Reynolds' relation $\frac{VD\rho}{\mu}$=constant. We take, therefore, for the equation of this part of the curve

$$\frac{Vd\rho}{\mu}=B, \text{ a constant at constant } L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The shape of the $L : V$ curve can perhaps be explained by introducing L in the form $f(d/L)$, writing $V=B'\frac{\mu}{\rho d}f\left(\frac{d}{L}\right)$ and expanding $f\left(\frac{d}{L}\right)$ as a linear function

$$V=B_2\frac{\mu}{\rho d}\left(1+C\frac{d}{L}\right)$$

since L is large compared with d.

$$V=B_2\frac{\mu}{\rho d}+C_2\frac{\mu}{\rho L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

whence we see that, *ceteris paribus*, LV is a constant. This is a hyperbola like that actually obtained, and this equation can be used to reduce the negative parts of the curves to the same line just as (1) was used for the 1st part.

The Upper Critical Points.—Analysis of the proceeding results, given in Part I, indicates that the upper critical velocity is dependent on both surface tension and viscosity, and can be represented by equation (3). Further evidence of the truth of this equation is given by plotting on the same graph (Fig. 5) the additional critical values obtained in this second part from the last two columns of Table C. It will

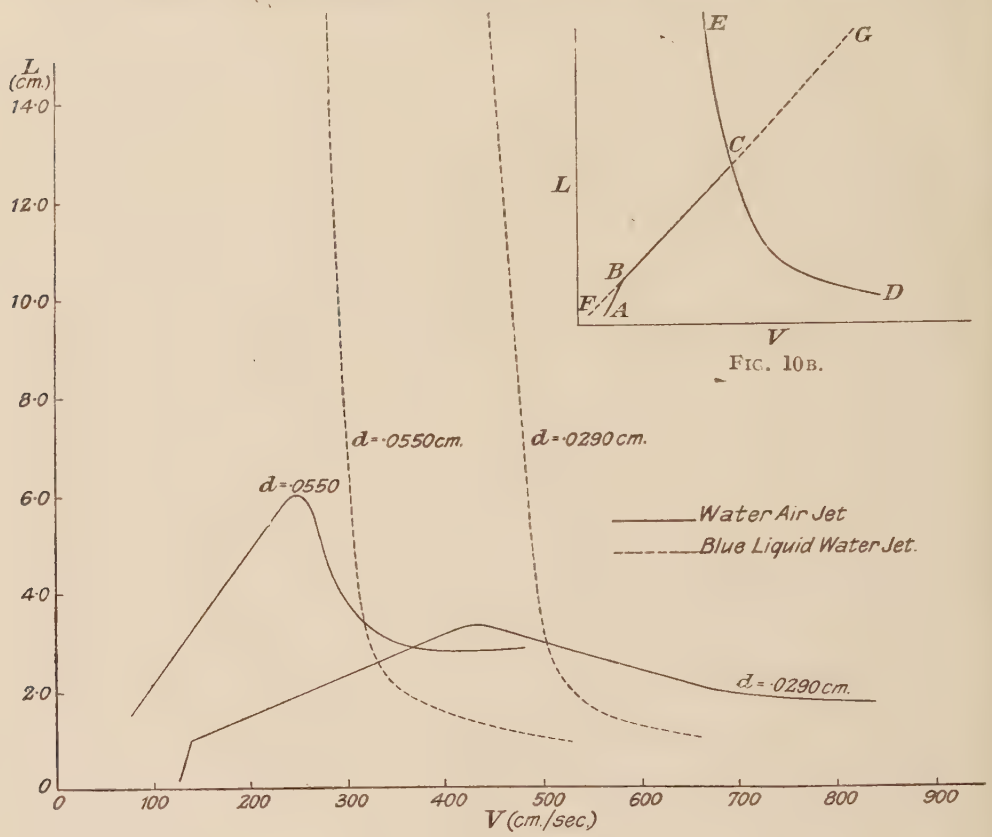

Fig. 10B.

Fig. 10A.

be seen that these results also lie on the straight line given by equation (3). This equation can now be written thus:

$$V_2 = A_1\sqrt{\frac{\sigma}{\rho d}} + B_1\frac{\mu}{\rho d} \quad \ldots\ldots\ldots\ldots \quad (3a)$$

and this form of the equation suggests a combination of equations (1) and (4). In order to illustrate this point further, we introduce the function $f(d/L)$ thus

$$V = A_2 f_1\left(\frac{L}{d}\right)\sqrt{\frac{\sigma}{\rho d}} + B_2 f_2\left(\frac{d_1}{L}\right)\frac{\mu}{\rho d} \quad \ldots\ldots\ldots \quad (5)$$

If, now, viscosity has no influence, B_2 must vanish, and

$$V = A_2 f_1\left(\frac{L}{d}\right)\sqrt{\frac{\sigma}{\rho d}}$$

putting $f_1\left(\frac{L}{d}\right)=\frac{L}{d}$ simply we get equation (1) of the positive part of the curve (in Fig. 5 the intercept on the "y" axis represents A_2L/d).

On the other hand, if the influence of surface tension is negligible, (5) becomes, putting $A_2=0$,

$$V=B_2f_2\left(\frac{d}{L}\right)\frac{l'}{\rho d}$$

which is equation (4) of the negative part. Equation (5) is then the *general equation* of the whole curve, excepting the small part below the lower critical point.

Anomalies when the Jet-length is Small.—Measurements of photographs of jets show that there is a sudden contraction at the orifice as the liquid (mercury) leaves the nozzle. This fact—and in addition the observation that, when the velocity of efflux is small, the continuous portion of the jet is traversed by ripples—can perhaps explain the abnormally low values of L when V is small—i.e., before the lower critical velocity V_1 is reached. These ripples, reacting on the nozzle, add an additional source of disturbance, tending to break up the jet sooner than if they were unable to reach the nozzle. A further cause of irregularity is to be looked for in the drop formation taking place very close to the nozzle; this may also cause a reaction at the nozzle tending to bring about disintegration in an unusually short time.

At higher velocities, on the negative part, a distinct "neck" was formed just outside the nozzle, and the resulting convergence of the stream between the nozzle and the "neck" no doubt prevents turbulence reaching the nozzle, even at the highest heads employed; L never again becomes zero. This reminded us of the similar experience of Osborne Reynolds in his experiment.

Summary.

The general phenomenon consists, therefore, of three distinct effects, the $L:V$ curves consist of three characteristic curves or phases superimposed:

(1) A straight line relation between L and V, the disruption involving surface tension effects, except at small lengths, when

(2) L/V is increasing due to drop-formation near the jet, and

(3) a hyperbolic relation between L and V, the breaking-up being due to turbulence caused by viscosity.

Elimination of either surface tension or viscosity would leave effect (3) or (1) respectively alone. The former has been practically eliminated in the experiment described above. To eliminate the viscosity effect would require a liquid possessing properties which no actual fluid has, falling freely into air, so that such conditions are unattainable. Again, the elimination of the initial phenomena at the jet should make (1) continue as a straight line to the origin.

These theories are summarised in Fig. 10B, which incorporates a $L:V$ curve *ABCD* with the corresponding observed continuation into the surfaceless phase *CE*, together with the theoretical continuation of the pure surface tension curve *FBCG*. Both surface tension and viscosity would effect disruption at a definite point in the jet. The actual length of a jet measured in air is determined by that one of these two properties which produces its effect first. At the upper critical point they would, operating alone, each break up the jet at the same place.

We are much indebted to Professor S. W. J. Smith, F.R.S., of Birmingham, both for his help in the work described in Part I and also for kind assistance in compiling the Paper. We are also grateful to Professor A. W. Porter, F.R.S., for kindly interest and criticism during the latter part of this work.

DISCUSSION.

Mr. W. E. Benton (communicated) : With reference to this very interesting Paper, I should like to point out that some slight obscurity results from the use of dimensional equations in the way which the authors describe. Equation (3) cannot possibly be derived from equation (2). Professor S. W. J. Smith, in his original Paper, suggested that the exponent " x " may, in itself, be of interest. From the experimental fact that, in the cases examined, " x " appeared to approximate to the value 0·5, he concluded that in these the influence of viscosity was at most very small. In the present work, the authors have assumed that the effect of viscosity is small, and on that assumption have written down equation (3), which can only be an expression of the possibility that the ratio of V_2 to the quantity $\sqrt{\frac{\sigma}{\rho d}}$ (which is also a velocity) is of the form $\left(A+B\frac{\mu}{\sqrt{\sigma\rho d}}\right)$. This involves the further assumption that μ only appears in the equation as part of a non-dimensional parameter. Both assumptions, however, appear to be justified by the experiments. It might further be suggested that there seems no justification for the broad assumption contained in the equation

$$L=K\rho^x d^y \sigma^w V^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{I})$$

Since $\sqrt{\frac{\sigma}{\rho d}}$ is of the nature of a velocity, it is possible that experimental evidence would support an equation of the form

$$\frac{l}{d}=Kf\left(V\sqrt{\frac{\rho d}{\sigma}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{II})$$

Equation (1), however, gives

$$\frac{l}{d}=K\left(V\sqrt{\frac{\rho d}{\sigma}}\right)^n$$

which is only a particular case of (II).

It appears from Fig. 3 that their equation (I) is only true for the methylated spirit curve. The corresponding portions of the water curves, when produced, do not pass through the origin. The curves for mercury appear to be parabolic.

Prof. S. W. J. Smith (communicated) : I rather regret that more of the excellent photographs of the jets obtained by Mr. Tyler, when a student in Birmingham, were not submitted for reproduction on a larger scale than those of the Plate which the Paper contains.

I had hoped also that Mr. Tyler would be able to include in the Paper some of the interesting results which he obtained, when working in my laboratory, on the influence of a magnetic field upon the form of the " characteristic curves." These experiments were begun by myself and Mr. Moss in the hope that they would throw further light on the significance of the results obtained below the lower " critical velocity," but circumstances did not permit us to complete them.

Mr. R. S. Maxwell (communicated) : The results obtained for mercury show a marked dissimilarity from those for the other liquids, particularly in what is termed the " positive " part of the curve. That this part is distinctly non-linear may be seen in Figs. 1 and 3. A possible explanation of this discrepancy may lie in the fact that mercury has a density of about thirteen times that of any of the other liquids used, whose densities are very nearly that of water. It would be interesting if experiments could be made on a liquid whose density is about half that of mercury for this would afford a check on the theory given. As there are not many suitable liquids, it would probably be necessary to work with molten metals, though in this case the expermental difficulties would be considerable. Fig. 4 shows that the position of the upper critical point depends upon the convergency or divergency of the nozzle. In Fig. 5 the results of a large number of observations, on different liquids, and by different observers, are collected

together, and the points are shown to lie approximately on a straight line. Were any precautions taken to ensure that cylindrical nozzles were used in all these experiments, as otherwise the positions of the critical points would be inaccurate ?

AUTHORS' reply: We are very grateful for the remarks of Mr. Benton pointing out the importance of the assumptions necessary in obtaining the dimensional equation 3 in the Paper.

We see no reason why, however, equation 3 cannot be derived from equation 2 when a knowledge of n is available, or, moreover, when μ appears in the equation as part of a non-dimensiona parameter.

His remarks on the non-dimensional quantity $\frac{\mu}{\sqrt{\sigma\rho d}}$ do not appear quite clear to us, unless he means that you cannot go back from $\left(\frac{\mu}{\sqrt{\sigma\rho d}}\right)^n$ to $f\left(\frac{\mu}{\sqrt{\sigma\rho d}}\right)$ unless μ only occurs in this non-dimensional expression, and in that he is right. The derivation of equation (II) given by Mr. Benton we regard as a process generally followed in evolving dimensional equations, that is from $f\left(V\sqrt{\frac{\rho d}{\sigma}}\right)$ to $\left(V\sqrt{\frac{\rho d}{\sigma}}\right)^n$, and it was not our intention to exhibit every step of the mathematical process.

Within experimental error the water curves in Fig. 3 do pass through the origin. The spirit curve is curtailed only because we could not use higher pressures, otherwise it is just like the water curves.

In reply to Prof. S. W. J. Smith's communication, it is hoped to publish shortly some interesting results on the influence of a magnetic field upon the form of the "characteristic curves."

The remarks of Mr. Maxwell referring to the discrepancy of the results between mercury and the other liquids, and attributing same to the excessive density of the former, are in agreement with the Authors' views. We have attributed the "positive part" of the LV curves to be an effect of surface tension alone—i.e., apart from viscosity. Hence, assuming any disintegrating disturbance at the nozzle capable of breaking up the jet in a constant time t, since observation shows that the diameter of the jet is uniform down to within a few mms., where it breaks up, the velocity remains constant down the jet.

So that if t=time of disintegration of the disturbance, which is carried down the jet with velocity V, the jet will break up after a length L, given by $V=L/t$—i.e., the $L : V$ graph is a straight line, or $V \propto L$.

Now, in the case of mercury, we suggest excessive density causes a continuous change of diameter down the jet, showing that V increases down the jet, so that L/V (V is now the velocity at the nozzle only) tends to increase as V increases. The order of "taper" required, however, does not appear on the photographs. It would indeed be of extreme interest to carry out further experiments using molten metals, and we are very grateful for this suggestion.

Mr. Maxwell quite rightly emphasises the importance of taking precautions to ensure that cylindrical nozzles were used, and to accomplish this condition the capillary tubing on being drawn out was cut at the point considered to give cylindricality (generally midway between points at which tapering began) and both halves of the tubing, each forming a nozzle, were experimented with and if the $L : V$ curves for both parts coincided, then this was sufficient to ensure that the nozzles were cylindrical.

XXVII.—AN INVESTIGATION OF THE CONTROL CONDITIONS UNDER WHICH NEWTON'S LAW IS VALID FOR THE EMISSION OF HEAT FROM ELECTRICALLY HEATED WIRES.

By Sybil Marshall, *A.R.C.S.*, and J. O. C. Vick, *A.R.C.S.* (Imperial College of Science and Technology, London).

Received May 21, 1925.

(*Communicated by* H. S. Gregory, Ph.D., D.I.C.)

Abstract.

The following Paper deals with the variation with temperature excess of the controlled heat loss from an electrically heated wire. The wire, whose temperature was measured by means of a Callendar-Griffiths bridge, was surrounded by air at a known pressure enclosed in a glass tube, the latter being immersed in a constant temperature bath. The rate of heat loss, therefore, could be calculated from a knowledge of the current used and the resistance per unit length of the wire. Total heat loss/temperature excess curves were plotted, these tending to become straight lines as the convection was reduced.

I. Introduction.

A PRECISE knowledge of the conditions under which Newton's Law is valid is essential in the determination of the thermal conductivity of metals, as may be seen from the following equation representing the thermal equilibrium of an element δx of an electrically heated wire surrounded by a gaseous atmosphere

$$KA\frac{d^2\theta}{dx^2}\cdot\delta x+\frac{C^2\rho_0(1+\alpha\theta)\delta x}{JA}=f(N, E, p, \theta)s\cdot\delta x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where K=thermal conductivity of the wire,
θ=temperature excess of element of wire above the surrounding atmosphere,
A=cross-sectional area of the wire,
C=current passing in the wire,
ρ_0=specific resistivity of the wire at 0°C.,
α=coefficient of increase of resistance with temperature of the wire,
δx=elementary length of wire,
s=perimeter of wire,
J=mechanical equivalent of heat.

The expression on the right-hand side of the equation, $f(N, E, p, \theta)s\cdot\delta x$, represents the loss of heat from the surface of the element, which in general depends on the nature, extent, pressure and temperature of the surrounding medium. Many investigators have, in order to simplify the theoretical aspect of equation (1), assumed

Newton's Law of cooling, by which the loss of heat from the element considered is proportional to its excess of temperature above that of the surrounding medium, i.e.,

$$f(N, E, p, \theta)s \,.\, \delta x = \lambda\theta s \,.\, \delta x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where λ is some constant.

A theoretical treatment of the cooling of bodies by both free and forced convection has been worked out by Oberbeck*, Boussinesq†, Lorenz‡ and Russell§, who have shown that Newton's Law is strictly true only for forced convection. Experimental observations have been made by Dulong and Petit‖, Macfarlane¶, Nichol**, Brush†† and others, on the rate of cooling of bodies of known specific heat, in order to determine their emissivity; whilst Schliermacher‡‡, Ayrton and Kilgour§§, Goldschmidt‖‖, Kennelly¶¶, Langmuir*†, King*‡ and others have obtained the emissivity by observing the loss of electrical energy from wires. These results confirm the predictions of theory.

The experiments described in the present Paper were undertaken in order to determine how far the emission of heat from electrically-heated wires could be controlled so as to conform to Newton's Law. The term "controlled emission" is here used to denote the heat loss from an electrically-heated wire, coaxial with a cylindrical tube whose external surface is maintained at a constant temperature (0°C.), the space between the wire and the wall of the tube being occupied by a gaseous medium.

In the case under consideration, if the temperature of the wire is uniform throughout its length, then $\frac{d^2\theta}{dx^2}$ is zero, and equation (1) may be written:

$$\frac{C^2R_{\theta}\,l}{J} = \frac{2\pi K_g\theta}{\log_e r_2/r_1} + \varphi(\theta, p, r_1, r_2) + \psi(\theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where C = current in the wire,
R = resistance of length l of wire at uniform temperature θ,
J = mechanical equivalent of heat,
K_g = thermal conductivity of gas surrounding the wire,
θ = excess temperature of wire above inner walls of tube,
r_1 = radius of wire,
r_2 = internal radius of tube.

The first term on the right-hand side represents the heat loss by conduction through the gas, from the wire to the walls of the tube, according to Fourier's Law. Maxwell has shown theoretically on the Kinetic Theory that this is independent of pressure down to a fairly low pressure, depending on the thickness of the gas layer, and recently this has been shown to hold experimentally by Gregory and Archer; the controlling factors in this expression then are the temperature excess of the wire above the wall of the tube, and the dimensions of the wire and tube.

* Ann. Phys., 1879, VII, 271.
† Comp. Rend., Vol. 133, p. 497.
‡ Ann. de Physik, 1881, XIII, 422, 582.
§ Phil. Mag., 1910, XX, 591.
‖ Ann. de Chim. et de Phys., 1817, VII.
¶ Proc. Roy. Soc., 1872, XV, 90.
** Proc. Roy. Soc. Edin., 1869, VII, 206.
†† Phil. Mag., 1898, XLV, 31.
‡‡ Wied. Ann., 1885, XXVI, 287.
§§ Phil. Trans., 1892, CLXXXIII, 374
‖‖ Elek. Tech. Zeit., 1908, XXIX, 886.
¶¶ Trans. Amer. I.E.E., 1909, XXVIII, 363.
*† Physical Review, 1912, XXXIV, 401.
*‡ Trans. Roy. Soc., 1914, CCXIV, 373.

The function $\varphi(\theta, p, r_1, r_2)$ denotes the heat losses by convection from the wire, which depend largely upon the pressure of the gas and the dimensions of the tubes, and to a less extent on the temperature gradient.

The function $\psi(\theta)$ denotes the heat loss by radiation from the wire, which depends only on the temperature difference between the wire and the walls, and not at all on the pressure or dimensions of the apparatus.

Thus at constant pressure equation (3) may be written:

$$\frac{C^2R_\theta/l}{J}=f(\theta) \quad \ldots\ldots\ldots\ldots \quad (4)$$

The aim of this investigation is to ascertain in what way the total emission depends upon the control conditions, and how these conditions can be fixed precisely so as to make $f(\theta)$ in equation (4) conform to Newton's Law.

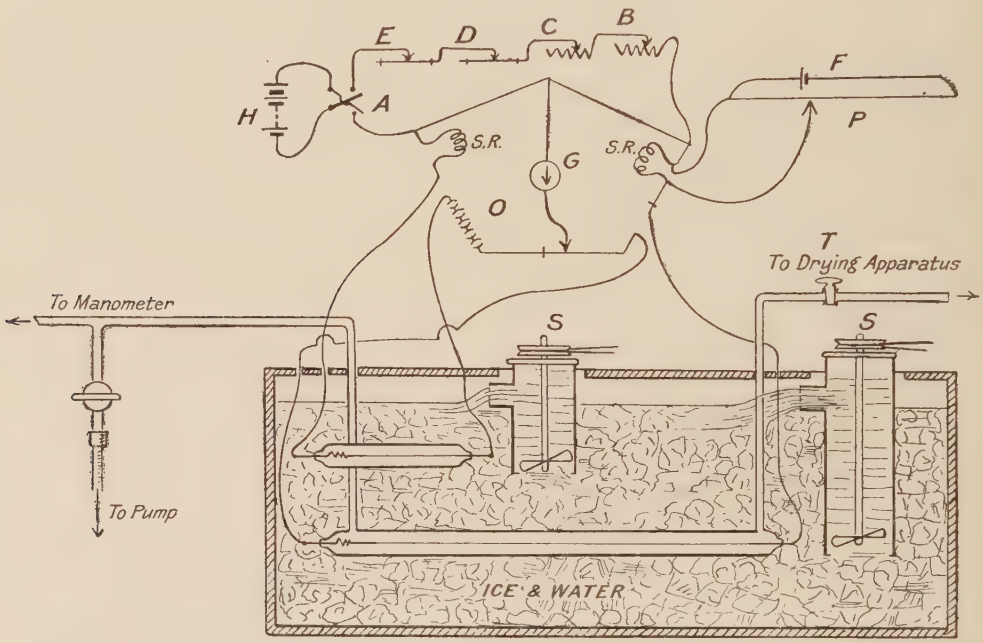

Fig. 1.—Apparatus and Connections.

II. Apparatus.

The apparatus was that used by Gregory and Archer in their determination of the thermal conductivity of air and hydrogen.

A diagram of the apparatus and connections is shown in Fig. 1. A long and a short tube of the same even bore were mounted horizontally and in gaseous connection. Down the axis of each tube was stretched a length of 6 mil platinum wire, to the ends of which were soldered leading wires of thick platinum sealed into the

tubes. One leading wire in each tube, in the form of a spiral spring, was slightly stretched in mounting, so that any extension of the wire on passing a current through it would be taken up, thus maintaining the wire coaxial with the tube at all temperatures. The two wires were placed in the main and compensating arms of the Callendar-Griffiths bridge O. The shorter tube or compensator was so arranged that it compensated for the heating effect in the leading wires, the loss of heat along the leading wires, and the thermo-electric effects produced in the main tube, in effect, isolating a length l of the wire, which was found from experiment to be equal to the difference in length of the two wires.

The mathematical analysis of this compensation is of too lengthy a nature to be here quoted, but is in course of publication by Gregory and Archer, to whom it is due.

A standard resistance SR of one ohm was placed in each of the opposing arms of the bridge, and the potential difference across one of these resistances was measured by means of the potentiometer P (Cambridge Scientific Instrument Co.'s "slide" type), F being a fairly large capacity accumulator. The current, supplied by the battery H, consisting of two sets of six 2-volt accumulators in parallel, entered the bridge, having passed through the commutator A and the sliding rheostats B, C, D, E, of which B and C were of the usual type, whilst D and E were stretched pieces of thick and thin manganin wire with sliders attached. By this means a very fine adjustment of current strength could be obtained. A moving-coil galvanometer G of a very sensitive type was used in the bridge circuit.

The tubes were immersed in a bath containing water, melting ice and snow, which were kept circulating and well stirred by means of S, S, two motor-driven stirrers. A Gaede pump was used to exhaust the tubes, whilst air, after having been passed over drying apparatus containing phosphorus pentoxide, could be admitted to the tubes through the tap T. A cathetometer microscope was used to measure the pressure as indicated by a mercury manometer.

The sliding contact on the Callendar-Griffiths bridge was fitted with a vernier, enabling readings on the bridge wire to be taken correct to 0·01 cms., so that since the resistance of the bridge wire was about 0·01 ohms per cm., readings could be taken correct to 0·0001 ohms, the fundamental interval being approximately 2·05 ohms.

The scale on the slide wire of the potentiometer was subdivided so that readings corresponding to differences of 0·0001 volts could be taken. Since these readings were of the potential difference across a standard 1 ohm coil, the readings of the potentiometer showed directly the current in amperes flowing through the coil and the wires in the tubes.

Two sets of tubes as described above were used, of radius 0·4592 cms. and 1·1714 cms., to show the effect of the alteration of dimensions of the tubes on the emissivity.

III. Experimental Procedure.

The coils of the two bridges used were first carefully calibrated in terms of the coils of highest resistance. The resistance of two measured lengths, each about a metre, of the platinum wire was obtained, immersed first in ice and then in steam. Pieces cut from these wires were then mounted in the tubes as already described.

A linear graph could then be drawn connecting the resistance per unit length of the wire with its temperature, from which the resistance for any temperature θ could be read off.

The above preliminary observations being taken, the apparatus was then connected up as shown in Fig. 1.

The bridge contact was set so as to correspond with a prearranged temperature, calculated from a knowledge of the fundamental interval for the tubes, and, having switched on the current, by adjustment of the resistances B, C, D, E, the current through the bridge and tubes was varied until the bridge was balanced, thus indicating that the wires in the tubes were at the desired temperature. Immediately, a potentiometer reading was taken to determine the current flowing in the circuit, and then another reading of the bridge galvanometer to make sure no change of current had taken place in the main circuit since last giving a balance.

Other settings of the bridge were taken, corresponding to various temperature gradients between 5° and 50°, necessitating currents of from 0·2 to 0·5 amperes. Series of readings were obtained at constant pressure, for pressures varying from 760 mms. to 60 mms., first with the narrow and then with the wide tubes.

IV. Results.

Dimensions.	*Narrow.*	*Wide.*
Internal radius of the tube	0·4592 cms.	1·1714 cms.
External radius of the tube	0·5215 cms.	1·2360 cms.
Radius of the platinum wire	0·007696 cms.	0·007696 cms.

The results obtained are tabulated below, the tables being made up as follows:—

Column (1) Series number and pressure in cms. of mercury at which the readings were taken.

Column (2) The temperature θ on the platinum scale as calculated from the bridge reading.

Column (3) The resistance per unit length in ohms per cm. at temperature θ as obtained from a knowledge of the fundamental interval.

Column (4) The current C in amperes as deduced from the potentiometer reading.

Column (5) Heat loss in watts per unit length of wire.

The readings for the wide tubes were taken with the bridge used for the narrow tubes for purposes of comparison.

NARROW TUBES.

(1) Series and Pressure	(2) Temperature θ	(3) R_θ/l	(4) Current C	(5) $\frac{C^2R_\theta}{l}$
I	40·3	0·06134	0·5137	0·01620
75·7	36·6	0·06060	0·4891	0·01448
	32·9	0·05985	0·4645	0·01293
	29·6	0·05915	0·4382	0·01135
	25·4	0·05825	0·4103	0·009825
	21·65	0·05732	0·3788	0·008240
	17·78	0·05670	0·3453	0·006775
	14·14	0·05597	0·3079	0·005315
	10·40	0·05520	0·2629	0·003818
II	45·95	0·06250	0·5438	0·01812
66·1	42·2	0·06170	0·5204	0·01679
	36·6	0·06060	0·4866	0·01436
	32·9	0·05985	0·4623	0·01280
	29·6	0·05915	0·43695	0·01129
	25·4	0·05825	0·4093	0·00977
	21·65	0·05732	0·3777	0·008175
	17·78	0·05670	0·3444	0·00674
	14·14	0·05597	0·3070	0·00527
	10·40	0·5520	0·2624	0·00381
III	45·95	0·06250	0·5417	0·01833
54·0	44·50	0·06222	0·5317	0·01763
	42·2	0·06175	0·5207	0·01676
	40·3	0·06134	0·5101	0·01595
	36·6	0·06060	0·4869	0·01433
	29·6	0·05915	0·4359	0·01127
	21·65	0·05732	0·3770	0·00816
	14·14	0·05597	0·3069	0·00527
	6·67	0·05445	0·2090	0·00238
IV	56·61	0·0647	0·5890	0·02243
24·85	45·95	0·0625	0·5397	0·01822
	42·2	0·06175	0·5191	0·01668
	36·6	0·06060	0·4847	0·01424
	32·9	0·05985	0·4606	0·01271
	29·6	0·5915	0·4350	0·01123
	25·4	0·05828	0·4078	0·00975
	21·65	0·05732	0·3764	0·00814
	17·78	0·05670	0·3434	0·00670
	14·14	0·05597	0·3064	0·005275
	10·4	0·05520	0·2616	0·00378

WIDE TUBES.

(1) Series and Pressure	(2) Temperature θ	(3) R_θ/l	(4) Current C	(5) $\frac{C^2R_\theta}{l}$
I	45·95	0·06250	0·5888	0·2168
75·6	42·2	0·06175	0·5635	0·01963
	38·5	0·06100	0·5372	0·01760
	34·8	0·06022	0·5093	0·01567
	31·0	0·05945	0·4781	0·01367
	25·4	0·05825	0·4297	0·01082
	21·65	0·05732	0·3935	0·00890
	17·78	0·05670	0·3540	0·00712
	14·14	0·05597	0·3112	0·005425
	10·40	0·05520	0·2612	0·00378

The remaining readings tabulated below were taken with the bridge used for the wide tubes.

WIDE TUBES.—*Continued.*

(1) Series and Pressure	(2) Temperature θ	(3) R_θ/l	(4) Current C	(5) $\frac{C^2R_\theta}{l}$
II	39·18	0·0611	0·5440	0·01808
76·5	35·40	0·06035	0·5164	0·01598
	31·70	0·05958	0·4848	0·01402
	27·95	0·05881	0·4542	0·01214
	24·20	0·05804	0·4210	0·01030
	16·74	0·05650	0·3444	0·00670
	9·26	0·05495	0·2487	0·003395
III	39·18	0·06110	0·52415	0·01677
60·43	31·70	0·05958	0·4677	0·01303
	24·20	0·05804	0·4050	0·009532
	16·73	0·05650	0·3302	0·006160
	9·26	0·05495	0·23895	0·003160
IV	46·65	0·06261	0·5225	0·01710
32·6	42·9	0·06187	0·4993	0·01544
	39·18	0·06110	0·4768	0·01390
	35·4	0·06035	0·4530	0·01240
	31·7	0·05958	0·4280	0·01091
	27·95	0·05881	0·4020	0·00939
	24·2	0·05804	0·3724	0·00805
	20·5	0·05730	0·3426	0·00673
	16·74	0·05650	0·3105	0·005295
	13·0	0·05573	0·2732	0·004165
	9·26	0·05495	0·2323	0·002963
V	48·5	0·06305	0·5040	0·01602
11·56	46·65	0·06261	0·4943	0·01530
	42·9	0·06187	0·4752	0·01397
	39·18	0·06110	0·4558	0·01269
	35·4	0·06035	0·4356	0·01144
	31·7	0·05958	0·4138	0·01021
	27·95	0·05881	0·3886	0·00888
	24·2	0·05804	0·3638	0·00769
	20·5	0·05730	0·3365	0·00649
	13·0	0·05573	0·2708	0·004085
	5·52	0·05388	0·1806	0·00176
VI	48·5	0·06305	0·5029	0·01597
6·36	44·78	0·06225	0·4834	0·01465
	39·18	0·06110	0·4549	0·01264
	35·4	0·06035	0·4346	0·01140
	31·70	0·05958	0·4132	0·01018
	27·95	0·05881	0·3886	0·00889
	24·20	0·05804	0·3638	0·00769
	20·50	0·05730	0·3364	0·00649
	13·00	0·05573	0·2708	0·004085
	5·52	0·05388	0·1810	0·001767

In Figs. 2 and 3 the heat loss per unit length of wire in watts (column 2) is plotted against the temperature, platinum scale (column 5), for the narrow tubes at

the different pressures taken. For purposes of comparison, Series I for the wide tubes was also plotted in the same diagrams. Fig. 4 gives the corresponding curves for the wide tubes.

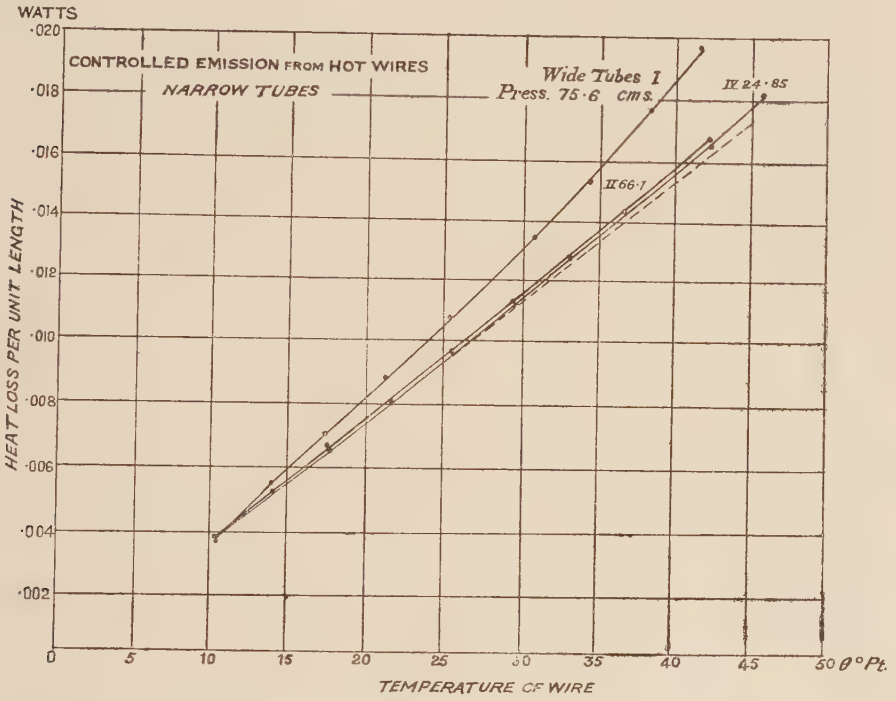

FIG. 2.

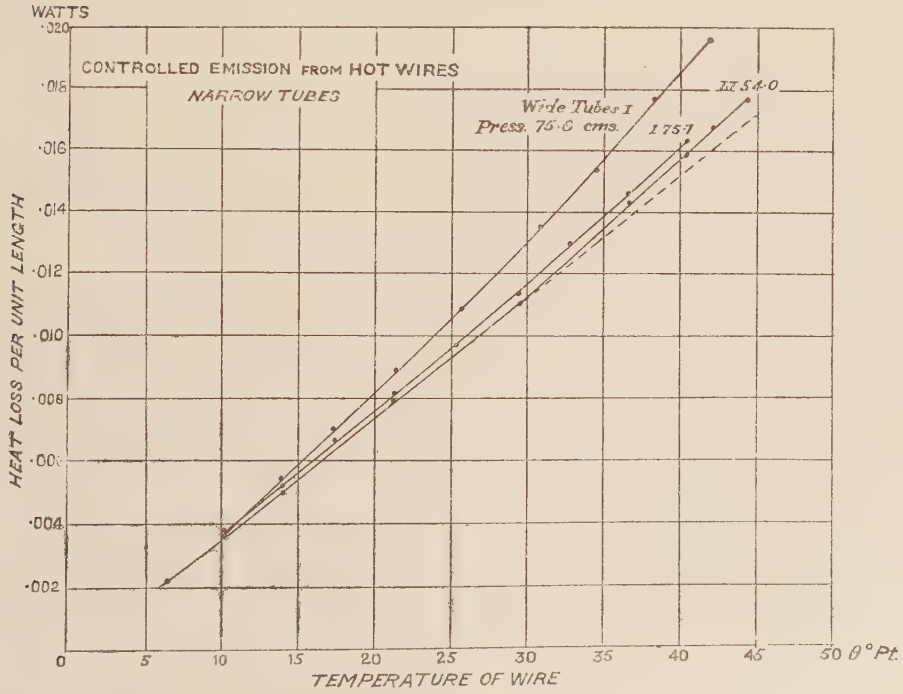

FIG. 3.

V. Discussion and Interpretation of Results.

In these results no attempt is made to separate the temperature drop across the glass from the temperature drop across the gas, between the wire and the wall of the tube. This has been done for two reasons. Firstly, because from the very nature of the problem of controlled emission of this kind there must always be a solid boundary of finite thickness across which there will be a temperature gradient. Secondly, it can be shown that for the material used (Whitefriars Flint Glass), with air as the conducting gas, the drop across the glass is only about one-thousandth part of the drop across the gas for the narrow tubes and half this value for the wide, a quantity which can have no appreciable effect on the curves obtained.

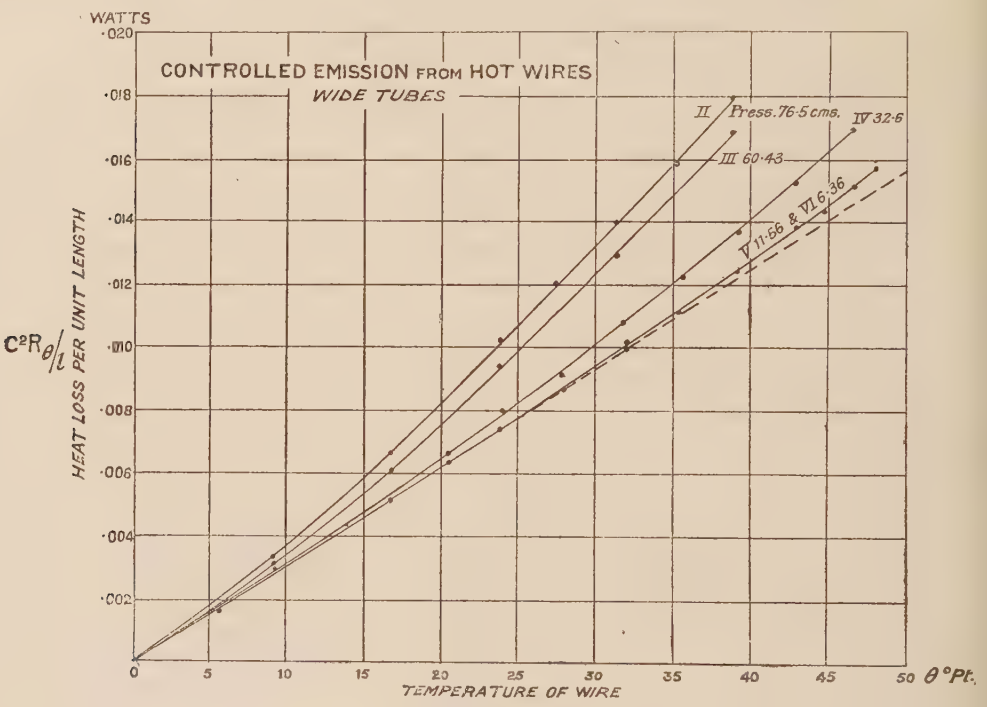

Fig. 4.

Sets of curves have been drawn showing the variation of the wattage heat loss per unit length of wire with the temperature, at constant pressure, for the wide and the narrow tubes, the dotted line being a continuation of the straight part of the low-pressure curves.

It will be observed that these curves tend to coincide for the lower pressures, but as the pressure is raised they separate and become increasingly convex to the temperature axis. The separation is due to the appearance and growth of convective losses at the higher pressures, since both conduction and radiation are here independent of pressure. The appearance and increase of convexity at the higher temperatures, which introduce a separation between the curves even at the lowest pressures, are due to convective losses and to the slight deviation of the conductivity

from a linear relation with the temperature. As the lower parts of the low-pressure curves coincide, one infers that, at these temperatures and pressures, convection, being the only factor depending on the pressure, has vanished, so that under these conditions conduction and radiation alone determine the shape of the curves. It might be pointed out that for these low pressures the curves are straight lines over a fair range of temperature, showing that, the radiation losses at these temperatures being small, the conduction of heat is here practically linear with the temperature.

The results for the two sets of tubes are similar, those for the wide tubes emphasizing the above tendencies. For the narrow tubes the curves approximate to straight lines up to much higher pressures than is the case for the wide tubes, showing that the deviation from Newton's Law increases with the radius of the tube. From this it would appear that the validity of Newton's Law is contingent in a large measure upon the absence of convection.

This work as a whole indicates that the emission of heat from electrically heated wires can be controlled so as to conform with Newton's Law by minimizing the convective losses. This may be brought about by using—

(1) Suitably small temperature gradients (e.g., for tubes 1 cm. in diameter containing air at half an atmosphere pressure the temperature gradient should not exceed 20°C.).
(2) Tubes of small diameter.
(3) Pressures as low as the dimensions of the tubes will permit, remembering that the conduction is no longer independent of pressure when the mean free path of the molecules of the gas becomes comparable with the radius of the tube.

In conclusion, the authors wish to express their gratitude to Dr. Gregory and Mr. Archer for the loan of the tubes and for much valuable help and advice received during the course of the investigation.

DISCUSSION.

Dr. J. S. G. Thomas: It may be of interest to mention the results obtained by several previous experimenters relating to the loss of heat from wires, tubes, cables and plane surfaces so far as these losses are dependent on the excess of temperature θ of the surface over the atmosphere. These can be represented by empirical formulæ as follows: Peclet, $a\theta$; Porter, $a\theta$; Eason, $a\theta^{1.23}$; Goldschmidt, $a\theta$; and $a\theta^{1.25}$; Kennelly, $a\theta^{1.1}$; King, $a(1+\beta)\theta$; Ayrton and Kilgour, $a\theta$. Langmuir's results show that in some cases the heat loss is proportional to the tenth power of the temperature of the heated wire. That convection and radiation losses are not dependent in the same manner on the excess of temperature of a wire over its surroundings can be easily demonstrated by determining the form of the current-temperature curves of a wire heated in either air or carbon dioxide in a wide or narrow tube. In the case of a narrow tube the temperature of the wire corresponding to any definite value of the current is always greater when the wire is surrounded by the CO_2 than when it is surrounded by air. In this case the heat loss from the wire is mainly conditioned by loss due to radiation. In the case of a wide tube, however, in which the heat loss is due principally to convection, for small values of the heating current the wire when in CO_2 attains a higher temperature than when it is surrounded by air, whereas for large values of the heating current the reverse is the case, the wire in air attaining a higher temperature than the wire in CO_2.

Mr. Rollo Appleyard: In all the experiments described by the authors the tubes were horizontal. It has long been known that convection effects depend greatly upon the angle with the vertical at which a hot bar or wire is held. With hot-wire ammeters and voltmeters, for example, accuracy requires that the instruments shall be read only when they are at the angle

at which they were calibrated. Before making general deductions from their measurements, therefore, it would be helpful if the Authors would repeat the investigation with the tubes at various angles.

Dr. ALEXANDER RUSSELL: I should like to point out that Newton only considered the problem of the convection of heat from a surface of a hot body immersed in a cooling blast of air. He considered the case of a block of iron cooled in a current of air flowing uniformly, and found that for a given wind velocity the heat carried away by the air was proportional to the difference of temperature between the body and the air before it was heated. In those days the melting point of ice, various kinds of wax, &c., gave a rough scale of thermometry. It has been verified by physicists that Newton's law is true within the limits of experimental error, even when the difference of temperature between the body and the air (before it is heated) is very great. It has nothing to do with Stefan's law, which states that the radiation of heat depends on the difference between the fourth powers of the temperature of the body and of the temperature of the cooling air, both temperatures being expressed in Kelvin's absolute scale of thermometry. Newton did not find how the amount of heat convected away varied with the velocity of the cooling draught; this was discovered by the French mathematical physicist Boussinesq at the beginning of the present century. Other things remaining the same, the rate of cooling depends on the square root of the velocity of the air; if you quadruple the velocity of the cooling blast twice as much heat will be carried away. There is a limit, therefore, beyond which it is uneconomical to increase the velocity of the blast. The rate of cooling is proportional to the temperature difference between the hot body and the cold air, as well as to the square root of the velocity of the blast, and lastly it depends on the shape of the body; Boussinesq has given exact solutions for cylinders, plates, spheres, &c. In many cases only a small error is introduced by neglecting the heat lost by radiation. It is interesting to remember that Prof. Kennelly found independently by experiment Boussinesq's solution for the convective cooling of a cylinder.

The CHAIRMAN, in calling upon Miss Marshall for a reply, said that it was not often that the Physical Society had the pleasure of listening to an authoress, and he felt sure those present would wish him to express the pleasure it gave them to have the opportunity of welcoming her on this occasion.

Miss MARSHALL thanked the Chairman for his kind welcome.

Dr. T. BARRATT (communicated subsequently): Newton's law was intended to apply to losses of heat by convection. It is therefore at first sight somewhat surprising that the authors find "the emission of heat from electrically heated wires can be controlled so as to conform with Newton's law *by minimising the convection losses*. Experiments by the writer (Proc. Phys. Soc., 28, I, December, 1915) showed that the heat emission from thin wires (in a wide metal enclosure) was composed almost entirely of convective losses. It is of interest, therefore, to examine the figures given in the present Paper for indications of relative losses by convection and radiation (conduction down the wires being very neatly eliminated). Their wire had a surface area per 1 cm. length of about 0·05 sq. cm. L. V. King (Phil. Trans., 373, November, 1914) quotes a formula by Lummer and Kurlbaum for the radiation loss e from a platinum surface, $e=0{\cdot}514\ (\theta/1000)^{5.2}$ watts per sq. cm. per 1°C. excess. This gives, for the wire used by the Authors, $e=0{\cdot}0003$ watt per cm. length per 10°C. excess. Their total loss per 1 cm. length for 10°C. excess is given as 0·003 watt, so that 90 per cent. of their loss is by convection for small excesses of temperature. The authors find (as everyone does) that as their wire is raised in temperature the total loss per square centimetre increases. This is accounted for by the rapid increase in the radiation portion of the losses, which follows approximately the Fourth Power Law even for bright platinum. The authors assume that the temperature of the inner surface of their enclosure remains at 0°C. It appears doubtful, however, whether the glass would be able to carry away at a sufficiently rapid rate the amount of heat brought to it from the wire. This would be particularly evident at the higher temperatures of the wire. The authors find, moreover, that Newton's Law appears to be more nearly followed in the case of the narrower enclosures. This would naturally be so if the foregoing explanation is correct, since the narrow tube would be more strongly heated by the hot currents of air. It is even possible that a limiting diameter of tube might be found where the increase in watts per 1°C. excess would be just balanced by the increase in temperature of the glass. It would be an interesting experiment to stretch a linear platinum thermometer along and in contact with the inner surface of the glass enclosure to find whether it remains accurately at 0°C. as the temperature of the central wire is raised.

AUTHORS' reply: Dr. J. S. G. Thomas' curves for air and CO_2 we find particularly interesting, since their form is similar to our own, and also because we have recently completed an apparatus suitable for the measurement of heat transmission through CO_2. Dr. Thomas' results point to the conductivity of CO_2 being smaller than that of air, which fact agrees with the existing values for the conductivities of air and CO_2, and they also point to the convective losses being greater for CO_2 than for air. This latter conclusion we shall endeavour to verify with our apparatus.

Mr. Rollo Appleyard may be interested to hear that in our new apparatus the wires and tubes are vertical. We thank him for his suggestion of using various angles of which we shall probably avail ourselves after having examined the vertical distribution.

(To Dr. Alexander Russell.) Though Newton originally formulated his law for forced convection, several experimenters have assumed it to apply to the mixture of conduction and free convection which occurs in an originally still atmosphere. For the law to be obeyed, the relation between the total heat loss from the body and its temperature excess must be rectilinear. Our experiments were carried out in order to determine the circumstances in which this type of relation holds for an originally still atmosphere, and they have shown that this occurs only for particular values of the temperature of the wire and the pressure and extent of the surrounding medium.

(To Dr. T. Barratt.) The reply to Dr. Russell's remarks applies also to the first part of Dr. Barratt's. In an otherwise still atmosphere the heat loss is due to a mixture of radiation, conduction and convection. The conduction seems to follow a parabolic law, which, for the temperature gradients used in the foregoing experiments, is found to be practically rectilinear. This parabola is concave to the temperature axis. The curves we obtained are convex to the temperature axis where convection predominates; therefore, by limiting convection, one can obtain a linear relation between the total heat loss and the temperature excess of the wire.

The radiation losses were deduced from the experiments of Gregory and Archer, who used the identical apparatus. Their results are in the course of publication by the Royal Society. They determined the heat loss from the wire at various temperatures *in vacuo*, and, after allowing for the "residual conductivity," they found that the radiation heat loss was a linear function of the temperature between 8°C. and 32°C. Their results show that the radiation heat loss at 32°C. is 0·0000769 watts per cm.—i.e., that it is 0·64 per cent. of the lowest value of the total heat loss at this temperature. At 8·4°C. the radiation heat loss is 0·00000925 watts per cm.—i.e., 0·31 per cent. of the total heat loss. This shows that the radiation heat loss is negligible for the scale to which the total heat loss curves have been drawn.

We have not assumed that the temperature of the inner wall of the tube is at the temperature of the bath. In Gregory and Archer's Paper on the conductivity of air it has been shown that there is a temperature gradient across the glass wall. This drop of temperature across the glass wall has been calculated from a knowledge of the conductivity of the glass used in the experiment, and, as we mentioned in the first paragraph of Section V of our Paper, we found that, for the thickness of the glass used, the temperature drop across the wall is negligible compared with that across the gas.

XXVIII.—RECENT RESEARCHES IN POSITIVE RAYS.

The Tenth Guthrie Lecture, Delivered April 24, 1925.

By Prof. W. Wien.

LADIES and Gentlemen,—I thank the Physical Society heartily for their invitation to deliver the Guthrie Lecture of the present year. I much appreciate the honour of this invitation, and hope that the progress of science, which cannot be effected by one nation alone, may increasingly be served by the co-operation of scientific men of all nationalities.

In giving you a short account of the progress of our knowledge on the subject of positive rays, I must explain at the outset that I shall not attempt to cover the whole ground. I shall leave on one side those experimental researches

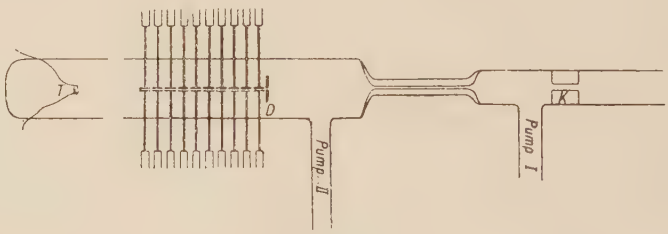

FIG. 1.

which are concerned with isotopes, because you in England are more familiar with that subject than we are in Germany. You will, therefore, pardon me if I confine my remarks to experiments which have been carried out in my own laboratory, and which, therefore, are more familiar to me than experiments my knowledge of which is derived from published accounts. All the admirable researches on isotopes suggested by Sir J. J. Thomson's work and carried out by Dr. Aston lie outside the scope of this lecture.

The first object of our experiments was the investigation of the conversion of charged particles into uncharged, and *vice versa*, which takes place in the positive rays owing to collisions with gas molecules. The theory of these changes is very simple. We assume that an uncharged particle moves along a certain path without any collision and then by impact with a slow-moving molecule loses an electron, becoming positively charged. On the other hand, a positively charged atom in the canal stream takes away an electron from a molecule by collision, and then goes on in an uncharged state. The mean free paths, L_1, L_2, in these two states are not of the same magnitude, the mean free path, L_2, of the uncharged particles being greater than that, L_1, of the charged. It is easy to show that the ratio of the two mean free paths is equal to the ratio n_1^0/n_2^0 of the numbers of charged and uncharged particles in the steady state.

Fig. 1 shows my first apparatus for determining the free paths and the relative numbers of charged and uncharged particles. The stream of canal rays from the perforated cathode K, after traversing a capillary tube and a slit D, passes between the plates of ten small condensers, each of which is 1 cm. wide and is separated from its neighbour by a gap of 1 mm. If one of the condensers be charged all the charged particles are deflected and only the uncharged particles pass on undisturbed. The stream impinges on a thermopile T, which accordingly measures its kinetic energy. If afterwards the charged particles be deflected, the energy recorded falls off accordingly. If the potential difference applied to the condenser is large enough all the charged particles are eliminated, and in that case no further change takes place in the reading of the thermopile when the potential difference across the condenser is further increased.

If we charge the first condenser sufficiently all the charged particles can be removed from the stream, but by collision with the molecules of the gas fresh charged atoms are originated. These can be eliminated by charging up a further condenser, the number of particles so charged and eliminated depending on the distance between the two charged condensers. Using only one condenser we get at once the ratio

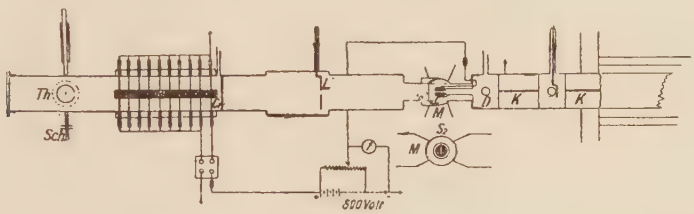

Fig. 2.

of charged to uncharged particles, and hence the ratio of the mean free paths. If we charge all ten condensers all the charged particles created throughout the path are eliminated, and the diminution of energy depends on the length of the row of condensers and on the mean free path of the particles in the uncharged state. This is the best method for determining the mean free path.

In this work I used ordinary canal rays composed of particles of mixed velocities and comprising atoms and molecules of different chemical elements. Rüchardt used a different apparatus which enabled him to study homogeneous pencils of particles, this apparatus being shown in Fig. 2. The canal rays pass through electric and magnetic fields at M, and the charged particles are spread out into parabolic sheets. The tube can be bent by means of a universal joint, S_2, and with the aid of a slit, L, in a zinc sulphide screen any desired homogeneous pencil can be selected.

Those rays of only one kind of atoms and the same velocity are going to the same system of ten condensers described in Fig. 1, where in the said manner both free paths are determined by means of a very sensible thermopile Th, designed by the Zeiss Works in Jena.

Rüchardt's results are very simple. We see, Fig. 3, that the reciprocal of the mean free path, L_1, when plotted against the pressure gives a straight line, but the latter does not pass through the origin as required by theory though the mean

free path must be infinite for zero pressure. But Rüchardt was able to show that this discrepancy arises from a residue of vapour in the discharge tube, which cannot be removed even by the most rapid Gaede pump. The effect could only be eliminated by using a glass vessel free from large masses of metal and from cement of any kind.

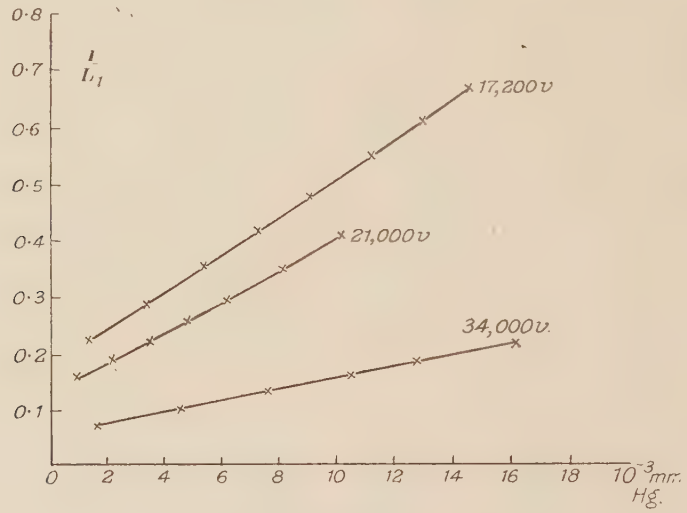

Fig. 3.

It is clear that from these substances vapour is always given off, so that a residual vapour pressure always remains where they are employed.

Fig. 4 shows the increase in the ratio of charged to uncharged atoms of hydrogen with the velocity of the particles as determined from the potential drop in the dis-

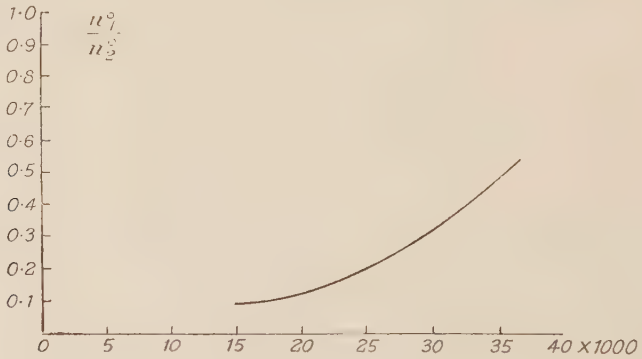

Fig. 4.

charge tube. A potential drop of 10,000 volts gives only about 10 per cent. of charged atoms, but 35,000 volts gives more than 50 per cent. It will be remembered that the ratio of the numbers of charged and uncharged particles is also the ratio of the mean free paths. In Fig. 5 we see how the reciprocal of L_2 depends on the velo-

city, L_2 being the mean free path for uncharged particles. L_2 decreases as the potential is raised. L_1, on the other hand, increases as the potential is raised (Fig. 3), that is to say as the speed increases, but its rate of change with velocity is greater than that of L_2.

The kinetic theory of gases gives a simple relation between the collisional radius of an atom and the mean free path. The volume of a cylinder whose length is that of the mean free path and whose radius is twice the collisional radius of the atom is equal to the reciprocal of the number of atoms per unit volume. The collisional radius so calculated is found to lie between the first and second of the Bohr orbits.

Rüchardt's measurements are fairly accurate. We can say that we know the mean free path of a hydrogen atom moving with the velocity of the positive rays to an accuracy of a few per cent.

Besides the mean free paths for charged and uncharged atoms, we have to consider other mean paths, connected with the light emission of the positive rays. The luminosity of these high-velocity atoms also arises from collisions with molecules of the gas in the discharge tube. We have, however, two kinds of collisions to consider, viz.: (1) Those which are responsible for the emission of light, and (2) those which destroy the emission of light from an excited atom. The latter I

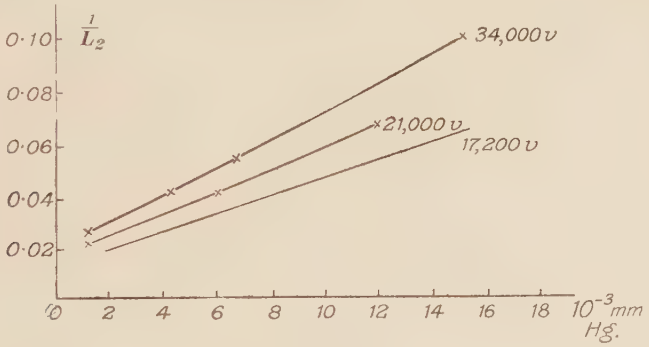

FIG. 5.

term "perturbations." Corresponding to these we have two kinds of mean path, and there is a third kind corresponding to the duration of luminosity in a pure vacuum. Consider first the collisions which are responsible for the emission of light. If we accept Bohr's theory, we suppose that in consequence of such a collision an electron is raised to a higher energy level, where it can stay for a time which measures the "duration of luminosity." Since the minimum amount of energy emitted by an atom is $h\nu$, we can calculate the number of these light-exciting collisions and the mean path if we know the absolute value of the light energy emitted by an atom per cm. of its travel. That value I determined eighteen years ago by comparing the light emitted by the canal rays with the radiation from a black body at a temperature of about 1,100°. The intensity of this radiation being known from the laws of radiation, we can calculate the energy emitted by the canal rays, and hence also the mean path of the excited atom—that is to say, the path which begins when the atom is excited by collision, and ends when the displaced electron returns to its normal orbit with the emission of a quantum of light. We car only find this mean path

by observations in a very high vacuum, when we are sure that there are no perturbations of the light emission by collision; for if collisions be permitted, a certain number of excited atoms will be charged by perturbing collisions before they have had time to emit light. In other words, the mean path for perturbations must be large, under the conditions of the experiment, in comparison with the mean path for the duration of luminosity. For hydrogen a vacuum of about ·001 mm. is sufficient. I was able to determine the duration of luminosity by employing an apparatus in

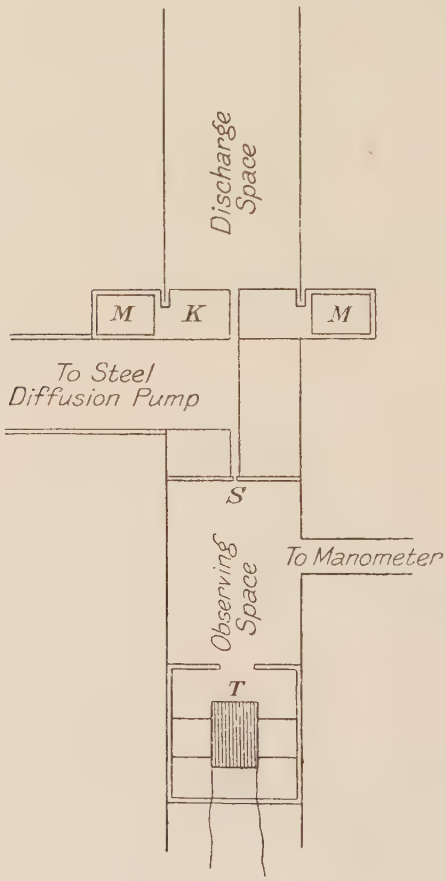

Fig. 6.

which the canal rays passed through a short, narrow slit into a region of high vacuum. For the strongest lines of H, O, N, He and Hg, I found that the intensity of light-emission decreases according to an exponential law with distance measured along the path of the rays, and from the exponential constant we can calculate the mean luminous path. In the case of the canal rays, this amounts to a few centimetres. If, however, we observe the luminosity of canal rays at ordinary pressures, we have to take into account all the complications arising from what I have called "perturbations." Observations made at various pressures give us the mean path for

FIG. 7.

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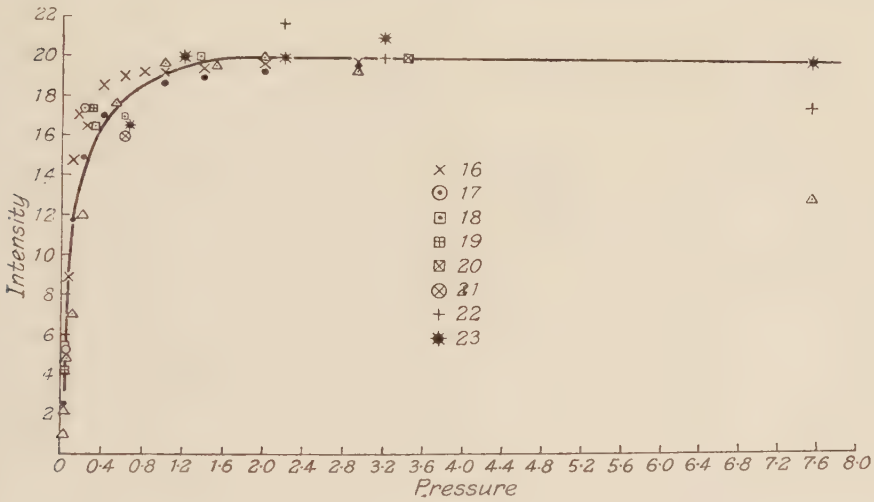
perturbations, that is to say, the path which begins when an average atom is excited by the raising of an electron to a higher energy level, and ends when the electron, by a perturbing collision, is removed from this level before the emission of light could take place. Recently I observed the luminosity of the canal rays up to a pressure of about 8 mm. of mercury. The apparatus is similar to the arrangement for the determination of the duration of luminosity, and is shown in Fig. 6. The canal rays on emerging from the slit S pass into the "observing space" at comparatively high pressure, while the region from which they emerge is kept at low pressure by a mercury-vapour pump. In this way it is possible to maintain a pressure of 8 mm. on one side of the slit and ·02 mm. on the other, so that the pressure on one side is 400 times that on the other.



$l=0$ $\lambda^\circ_1=60.$ $L_2{}^0=300.$ $l=0$ $\lambda^\circ_1+150.$ $L_2{}^0=300.$

FIG. 8.

Now the intensity of the line Hα (displaced by the Doppler effect) has been compared for various pressures, and it has been found that the intensity increases in proportion to the pressure in high vacua, but for pressures of the order of ·5 mm. the intensity becomes constant. It is easy to see that this behaviour results from the influence of perturbing collisions on luminosity. If there were no perturbations, the luminous intensity should increase in proportion to the pressure, because the frequency of light-exciting collisions increases at that rate. This is, indeed, the case at low pressures, where no appreciable perturbations take place, but at higher pressures the luminosity is diminished by collisions. With increase of pressure the mean path for perturbations becomes small compared with the mean luminous path, and the increase in the number of excited atoms is compensated by the increase in the number of perturbations. Fig. 7 shows the canal rays entering a region of 8 mm. pressure and disappearing after a short distance. Fig. 8 shows how the intensity becomes constant with increasing pressure. Fig. 9 gives a comparison of

experimental with theoretical curves: the agreement is quite good. In these figures L_2^0 is the mean path of the uncharged particles at a pressure of ·001 mm.; l the luminous path *in vacuo* on the hypothesis of a certain "life time" of excited atoms; λ the logarithmic decrement of the luminosity *in vacuo* in accordance with the wave theory of light; l^0_1, λ^0_2, are corresponding mean paths of perturbations.

It may be noted in passing that these curves answer another question. The classical theory explains the duration of luminosity by the damping down of vibrations owing to gradual loss of energy by radiation. On the other hand, the quantum theory suggests that the duration of luminosity is the statistical mean of the time which elapses between the excitation of an atom and the return of the disturbed electron to its normal orbit. It seemed not impossible that both effects might take

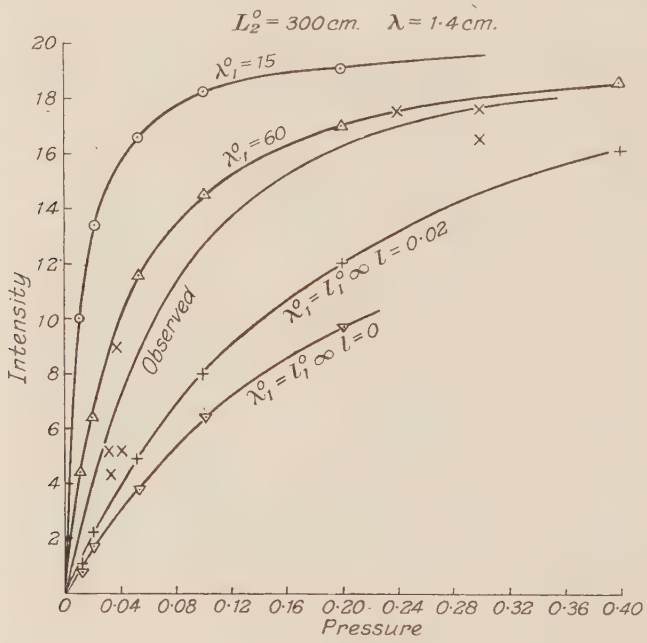


Fig. 9.

place simultaneously. In that case we should have two kinds of perturbations, one destroying the vibrations and the other removing the displaced electron from the atom before the emission of light can be effected. But if we have two independent causes of perturbations, the intensity of luminosity will ultimately be diminished more than it can be increased by increasing the number of light exciting collisions, and the curve will drop after reaching a maximum. The observations show, however, that the intensity reaches a constant value, so that there is only one kind of perturbations. Hence we have to choose between a gradual damping of the oscillations and a statistical duration of the excited condition of the average atom.

Using a modification of the apparatus designed for the measurement of the duration of luminosity, I have been able to discriminate between the spectral lines from charged and uncharged particles. The canal rays emerging from a slit into a high vacuum

FIG. 10.

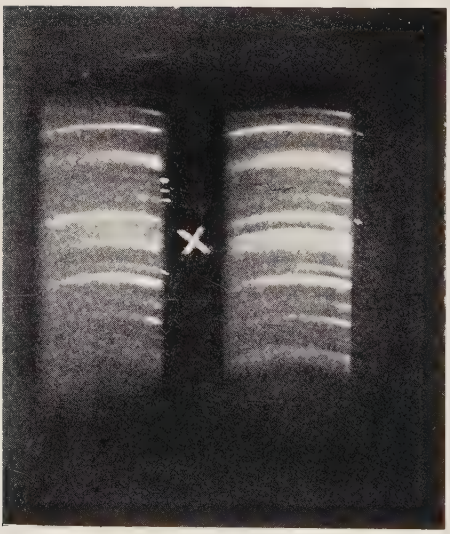


FIG. 11.

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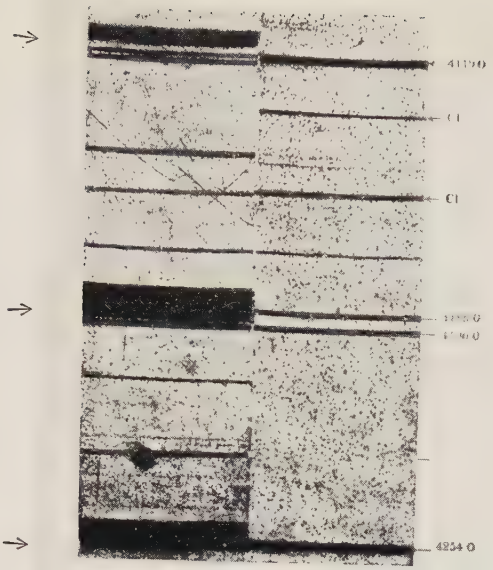

FIG. 12.

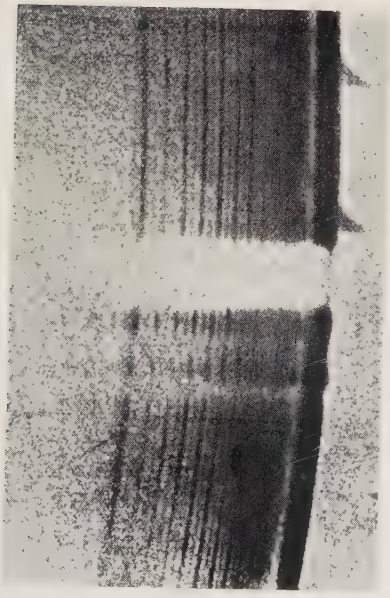

FIG. 14.

[*To face page* 331.

pass between the plates of a small condenser only 1 mm. wide. The condenser must be small and very close to the slit, because in the high vacuum the luminosity of the canal rays extends over only a short distance. In the high vacuum no appreciable changes take place in the charges on the particles ; hence, if we form an image of the pencil of rays on a photographic plate by means of a spectrograph and charge the condenser, we can make out which spectral lines are emitted by charged and which by uncharged particles : in the former case the lines are deflected, and in the latter they are not. Fig. 10 shows the O and H lines. The latter are not deflected, but some of the former show a very remarkable deviation. These lines, the well-known spark lines, are emitted by charged particles. In Fig. 11 the cross marks the position of one of the nitrogen bands. Comparison of the two photographs shows that in the case where the condenser is charged the band is displaced. Some of the N bands, the " negative " bands, are emitted by positively charged molecules.

In close connexion with the problem of luminosity is the paradoxical fact that the Doppler displacement reaches a limit with increase of the voltage accelerating the canal rays. Fig. 12 shows the Doppler displacement for spark lines of O, these lines being indicated by arrows. H lines have a relatively smaller Doppler displacement. But the arc lines of O also show a much smaller displacement, and we shall find that the difference arises from the fact that the arc lines are emitted from uncharged particles and the spark lines from charged particles. Electromagnetic methods have settled beyond doubt that the velocities of the atoms increase with voltage in the manner predicted by theory, but several physicists have agreed in finding that the Doppler shift reaches a certain limit beyond which it does not increase with increasing voltage. I have suggested several researches on this question. Dr. Krefft has raised the voltage of a discharge tube filled with hydrogen to 70,000 volts, but found no limit to the Doppler shift. The maximum velocity increases according to the square root of the voltage in accordance with theory, but he finds also that the absolute value of the shift is smaller than that which would be expected from theory. This difference arises from the fact that the H lines are emitted by uncharged atoms. The Doppler displacement of arc lines measures the velocity of the uncharged atoms, while the electromagnetic method measures that of the charged atoms, and the former have on the average a much smaller velocity than the latter, so that the greater part of the light is emitted by atoms of comparatively small velocity. The spark lines being emitted from the charged atoms we find in their case that the Doppler displacement is in agreement with the results obtained by electromagnetic methods. On the other hand, the arc lines for oxygen coming from uncharged particles show a small Doppler shift like the H lines. It seems to me, therefore, that the difficulties concerning the Doppler displacement have been cleared up.

An interesting method for the observation of the Doppler effect has been worked out by Rau. His apparatus is seen in Fig. 13. Close to the path A of the canal rays a glass cylinder is fixed with its axis perpendicular to the rays. The rays of light coming in all directions from the canal rays are refracted by the cylinder. If the cylinder be focussed by means of a lens L on the slit S of a spectroscope, although the axial rays will not show any Doppler effect, since they are normal to the direction of motion, yet, in addition to them, rays from approaching and receding particles will be deflected into the spectroscope, so that on the spectrogram we can get spectral lines with various Doppler shifts. Fig. 14 shows a band spectrum of nitrogen. In

the middle we see the cathode and images of the canal stream behind and in front of the cathode. On each side the Doppler effect is visible as a bend in the otherwise straight spectral line. Rau obtained in this way evidence of band spectra emitted by positively charged nitrogen molecules, and I was able to confirm this conclusion later by the method of deflection already explained. Rau also found by the same method, with very long exposures, that the second spectrum of hydrogen shows very faint Doppler lines. It is, however, much easier to detect curvature in a spectral line than to find the Doppler line among a great many other lines.

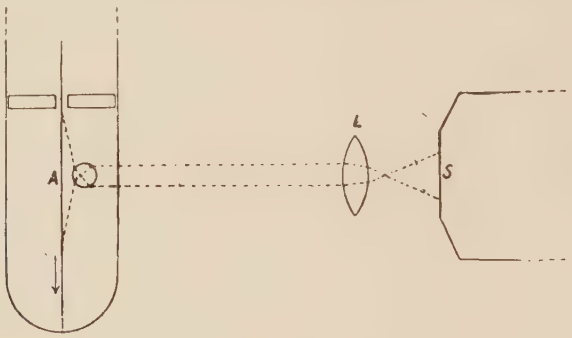

Fig. 13.

The electromagnetic analysis of canal rays using a thermopile has been further developed by Döpel. He worked with Krefft at a comparison of the velocities measured by electromagnetic deflection with those measured by the Doppler shift. Further, Döpel has compared the canal rays originated by tubes of different forms. Dr. G. P. Thomson has shown that a spherical chamber gives more H molecules in proportion to atoms than does the cylindrical type. But in spite of that the absolute number of molecules as measured with a thermopile is greater in a cylin-

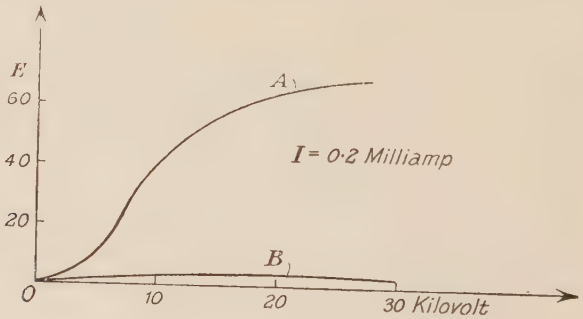

Fig. 15.

drical tube. Fig. 15 shows how great this difference is in the case of hydrogen, B showing the percentage of molecules (E) for a spherical chamber, A that for a cylinder. In England the spherical chamber is preferred. Perhaps this is to be attributed to the British love of moderation.

The electromagnetic analysis of positive rays by Döpel has given some interesting new results. Fig. 16 shows the parabolas obtained by means of simultaneous electric

and magnetic fields. Besides the well-known parabolas of H, H_2, H_3, He, Ne, with positive charges, negatively charged hydrogen molecules and helium atoms are found. It is important to know that negative He atoms can exist.

In almost all the parabolas there are two maxima of intensity at two different velocities. Sir J. J. Thomson suggests that this is due to discontinuities in the ionisation which takes place in the discharge tube, but the parabola of H_3 shows only one maximum, and it is difficult to understand why this molecule should be the only one free from such discontinuities. The position of the second maximum suggests the explanation that it results from molecules of double mass accelerated in the discharge tube, but dissociated before entering the canal stream. We must

FIG. 16.

therefore assume the existence of H_2, H_4, but not of H_6, since the parabola of H_3 does not show a second maximum. Attempts to detect H_2 and H_4 in the canal stream itself have not yet shown the supposed molecules, but I do not know what more probable explanation can be put forward for the occurrence of the two maxima.

You see that the deeper we penetrate into the behaviour of canal rays the more new problems we find. The study of these rays is of great importance for physicists, because it enables us to observe the atoms under very simple conditions. But all the new questions which are raised by each new observation require the co-operation of many observers, prosecuting the work with their united efforts. British and German physicists have both contributed their share to the study of positive rays, and I hope that their co-operation will be of long duration.

XXIX.—INTENSITY OF MULTIPLE SPECTRAL LINES: EXPERIMENT AND THEORY.

Lecture delivered 22nd MAY, *1925.*

By Prof. L. S. ORNSTEIN, Rijks Universiteit, Utrecht.

THE problem of measuring the intensity of spectral lines derives its importance and sense from the theory of Bohr. In former spectral investigations attention was paid to qualitative estimation of the intensity, and spectral lines were valued without any precision. It was generally believed that the intensity of spectral lines depended haphazardly on the circumstances of emission, and therefore a precise estimation of the intensity could be considered of no use. Bearing in mind that, according to the theory of Bohr, spectral lines are emitted if an atom passes from a state of higher energy to one of lower energy, it is clear that there must be cases where the intensity of a spectral line does not depend on external circumstances; for instance, if the electron falls from an energy-level a into one of the levels b_1 or b_2. The simplest case of this would be the emission of the 4-3 and 4-2 lines of hydrogen. The first cases we worked out were all of this type, and it was only afterwards that other cases were considered and measured.

It is my intention in this Paper to outline the experimental method followed by myself and collaborators and to give a discussion of the results obtained with it. Up till now all our measurements have been performed in the photographic way; and it is only very recently that we have begun to measure spectral emission with a quick and sensitive combination of a new thermopile and a galvanometer, constructed by Dr. Moll and Dr. Burger.

If we want to compare intensities with the aid of the photographic plate, we have to measure the density caused by the light on the plate; and we have to know the relation between density and intensity. The measurement of density can be effected by different methods; in our Institute the registering microphotometer of Moll was used, which was demonstrated to this Society some years ago.

Now, if with the aid of density intensities must be compared, there is a complication that radiations differing much in wave-length have to be compared, and, as we all know, the photographic plate shows a great difference of sensitiveness for light of different wave-lengths. That means that equal quantities of energy differing in wave-length working for an equal time on the plate cause a different density.

So our experimental method had to be worked out in two stages. First we had to develop the method in the case of spectral lines of nearly equal wave-length. Later on this method was perfected by the determination of the energy-sensitiveness of the plate.

I shall begin by describing the first method in its exactest form. I shall then proceed to indicate a simplification that is possible in this case; finally, for the experimental part of this paper, I intend to describe the method used in my Institute for comparing the intensities of distant spectral regions.

If the intensity of two spectral lines of about the same wave-length has to be

compared, the question can be put in this way: How much must the strongest line be reduced in order to be equal in strength to the weakest? If we have at our disposal a substance of which the absorption in the interval of wave-length used is known, we can proceed in the following way: The spectral lines to be compared are photographed with the spectrograph; and this operation is repeated, the absorbing substance being put before the slit of the spectrograph, until we get a plate where the density of the weakest line in the first photograph is the same as that of the

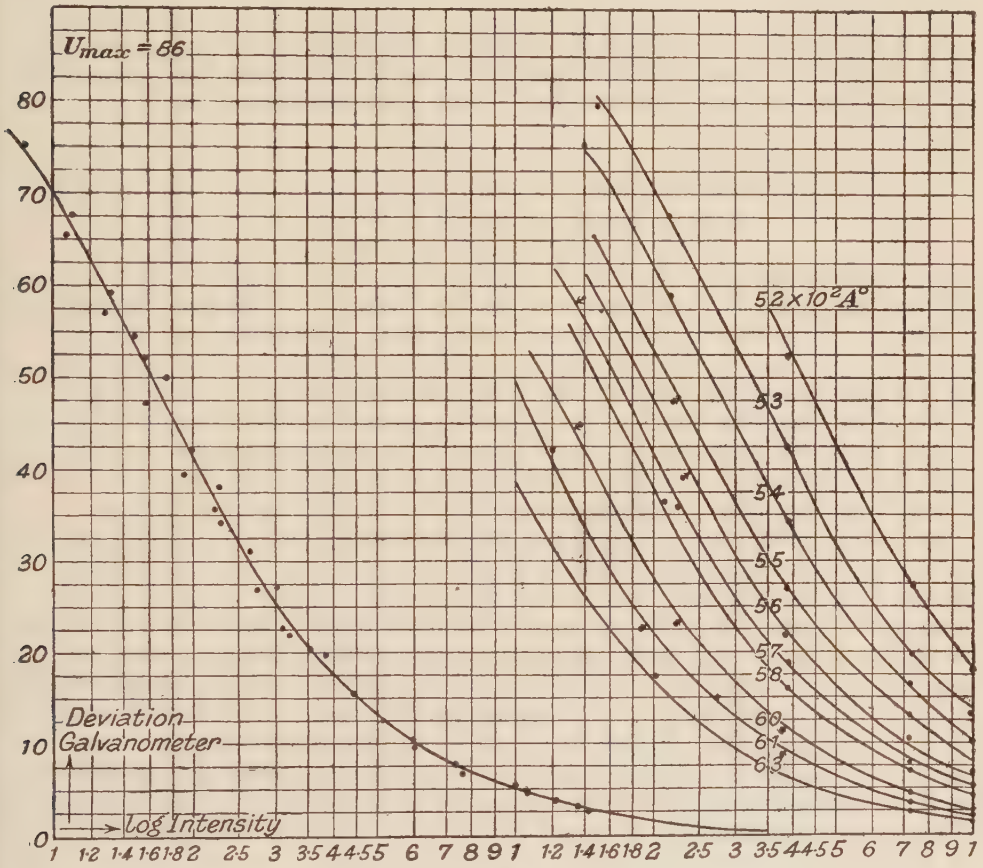

FIG. 3.

weakened image of the strongest. It is very troublesome to proceed in this way, and it is clear that it is not the precise weakening that equalises the two lines that has to be searched for, but that a method of graphical interpolation must be used. Both spectral lines had better be photographed on the same plate during the same time, with different intensities. Instead of making the exposures one after the other, it is possible to take them at the same time. We have only to take care that the image of the spectral line on the plate is everywhere of the same intensity, and to divide this image in parts of different but known strength by putting absorbing

substances in the optical system. Fig. 1 shows a photograph of the triplet of Mg 5183, 5172, 5167 ; 1p—1s. Fig. 2 shows Cd 5085, 4799, 4678, 1p—1s.

By simultaneously photographing the weakened and the unweakened spectra we have the advantage that we are independent of the variations of the sources of light.

Now, from the plates obtained in this way we can easily determine the relative intensities of the lines. For this purpose they have only to be photometered, the deflection through the non-blackened plates being measured at the same time. Thereupon a graphical representation of each of the spectral lines is made by taking the logarithm of intensity as the abscissa and $\frac{U}{U_o}$ as ordinate. In this way we get the following representation, where for every line the unweakened image has been put 100.

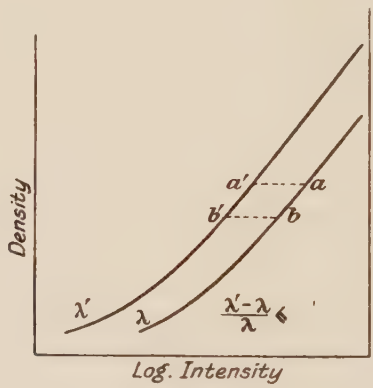

FIG. 4.

From this graphical representation the proportion of the intensity of the lines can be easily seen (Fig. 4). If a represents the density corresponding to 100 per cent. of the second line, the point a' on the density curve of the first line has the same density, and the percentage intensity of the second line may immediately be read off as the abscissa of the point a. If we compare the point b with the corresponding point b', we know what fraction b' is of the strongest line ; and as we also know what fraction b is of the weakest line we thus get a second measurement of the proportion. As is easily to be seen, the distance of the graphs on the logarithmic scale in both cases yields the proportion of the lines—i.e., the density curves are parallel. Better than by repeated measurements, the proportion of the lines can be determined by applying the parallelism of the density curves. This parallelism can be used in order to draw the curves as exactly as possible, and then to determine the distance with the greatest precision possible. In this connexion the remark may be important that, not only for narrow intervals of wave-lengths the parallelism can exist (compare Fig. 3) ; to a high degree it depends on the kind of plate and the mode of development used.

Before leaving the subject of this method of measurement, I should like to say a few words about the method of weakening used. Whereas for absorption-spectra

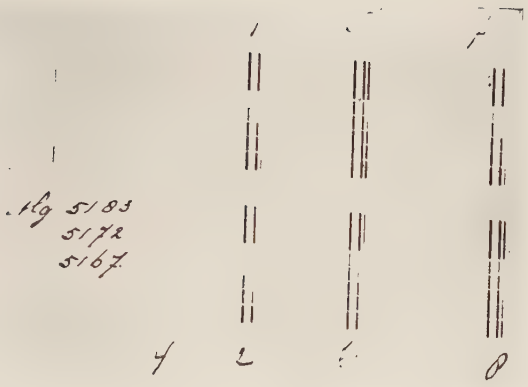

FIG. 1.

FIG. 2.

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the marks of density can be given by an echelon-form slit, or by any other form of a geometrical weakening of measurable diaphragms, this mode of operation is not practicable for line-spectra. In our researches we have made use of smoke-glass weakeners, adjusted on a lens, Le_1, which is homogeneously filled by the source of light. A second lens, Le_2, forms a sharp image of the weakeners on the slit Sp of the spectrograph.

In this way the plates demonstrated (Figs. 1 and 2) have been photographed. This method requires an astigmatic spectrograph. The demonstrated plates have been taken with a grating arrangement described by Van Cittert.

Now the smoke glass has to be standardised as to the fraction of light absorbed. This can be effected in two ways—

1. In the arrangement with a continuous source of light. The marks of intensity are given by variation of the width of the slit, the intensity for each wave-length being proportional to the width. In the way described above the density caused by the source of light when the weakeners are interposed gives a measure for the absorption.

2. The smoke glass can be standardised by direct measuring with thermopile and galvanometer, the weakeners being interposed in the beam of light produced with the help of a monochromator. This second method is simpler but less certain

Fig. 5.

for those cases where the loss of light is partially due to scattering or other causes besides absorption.

In the ultra-violet, Dorgelo (Philips) and Frerichs (Bonn) have used other methods of weakening, smoke glass not being applicable in this region.

Of late, investigations made by Dr. Burger and myself have shown that the whole method may be considerably simplified. In the beginning many precautions were necessary on account of the bad reputation of photographic methods as to precision. However, as on closer examination this bad reputation appeared to be undeserved, and rather exact results were to be obtained by a careful handling of the method, it became possible to proceed in a less complicated way. We were able to give the marks of density independently with another source of light, and it appeared to be unnecessary to take equal times of exposure for the spectrum under consideration and the density marks. In this way we have investigated the spark-spectrum of Al, Zn, Si, exposing the photographic plate for one or two seconds, and putting on the same plate, during 30 seconds, the continuous spectrum of an incandescent lamp with silicon window. This lamp yields sufficient intensity in the far ultra-violet for producing marks of density by variation of the width of the slit. In proceeding in this manner we found that measurement on multiple lines can be performed as exactly as was possible in the more complicated way described above. The proportion of the spectral lines appeared to be as well in accordance with the predictions as were the results for the visible part of the spectrum. The density curves are exactly parallel for the plates used, even for those wave-lengths which are

at a great distance form each other. Complications were only to be found in the region of gelatine absorption. In this connexion we may mention that also for the X-ray region the photographic method may be applied, as is demonstrated by Dr. Bouwers in his Utrecht dissertation.

An interesting feature, albeit of no great use for the application, is the fact

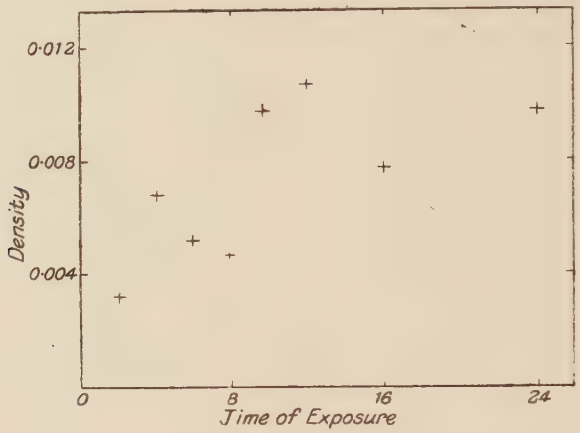

FIG. 6.

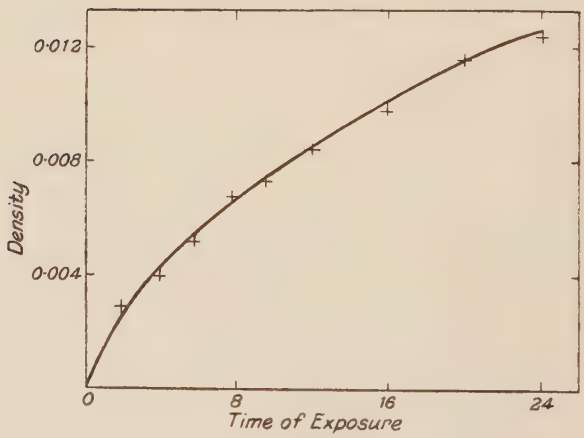

FIG. 7.

that in the visible as well as in the Röntgen region the density for plates developed without KBr may be represented by the formula

$$d=\log_{10}\frac{U_o}{U}=c\log.\left(\frac{it^p}{a}+1\right)$$

where c, p and a are constants, t is the time of exposure, and i the intensity of the light falling on the plate. Figs. 6 and 7 represent the density curves before and after a correction for the faults of the photographic plate has been applied to them.

Hitherto we have confined ourselves to the measurement of intensity for those

cases where the spectral sensitiveness of the photographic plate—i.e., the density caused by equal energy in equal time—is the same for different wave-lengths. It is a well-known fact this is no longer the case if the difference of wave-length is greater.

Fig. 8 indicates the density caused in equal times on a panchromatic plate by the energy of an incandescent lamp, the energy curve being given in the same diagram. The figure shows that the density is a very bad measure for energy, and that, therefore, the qualitative measuring of density in the spectrum has hardly any significance.

In order to compare the energy of more distant spectral lines, we have used a standardised lamp. As such we chose a common 4 volt nitra incandescent lamp

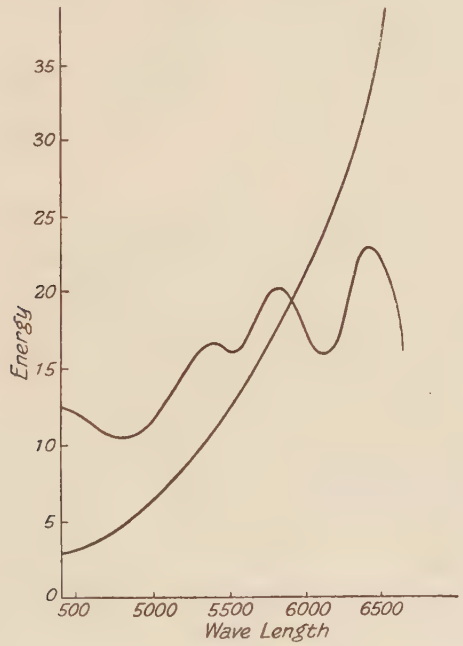

Fig. 8.

considerably under-run (2·5 v.), in order to obtain a longer duration of life. I will briefly describe the method of standardising the lamp. A monochromator has been used that was built up out of two identical spectroscopes in such a way that the spectrum formed by the first was folded together by the second on its own slit. Now, if we put a slit in the spectrum of the first spectroscope only that wave-length passes the instrument which falls on the intermediate slit.

The monochromator constructed by Dr. van Cittert excels by its strength of light and spectral purity. Now, in order to standardise a lamp with this monochromator we proceed as follows :—

Before the monochromator a strongly over-run lamp is placed, and by lenses an image of the lamp is formed in the point T. For the different colours corresponding to the different positions of the slit in the monochromator the intensity of

this image is measured by a thermopile. Thereupon the slit in the monochromator is removed, which causes a white image at the point P of which we know the spectral composition. By means of a lens, L_2, an image is formed of P on the slit of a spectrograph. Then at the point P the spiral of the standard lamp is placed, and again a photograph is taken. The strongest of the two images on the slit is weakened in

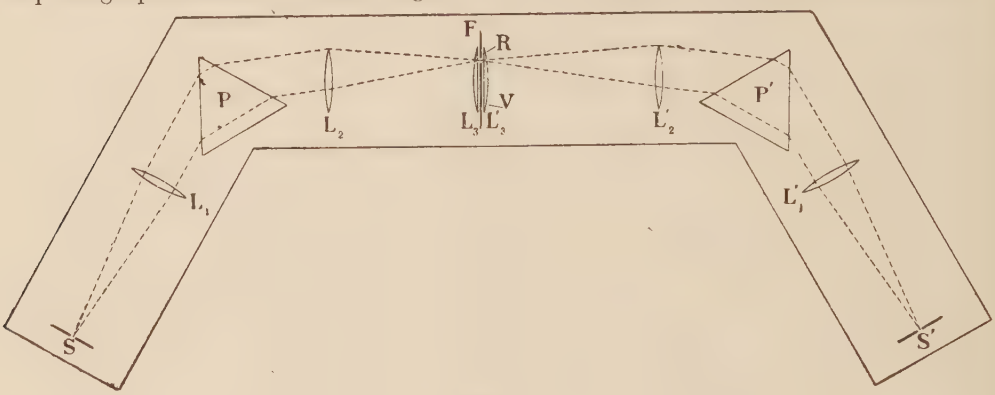

Fig. 9.

known proportion in order to get marks of density for comparison of the spectra of both lamps.

This comparison can be made for every wave-length. And, as in the image of the over-run lamp, the proportions of the energies are known, those of the unknown lamp can in this way also be determined. Fig. 8 shows the emission curve of a standardised lamp.

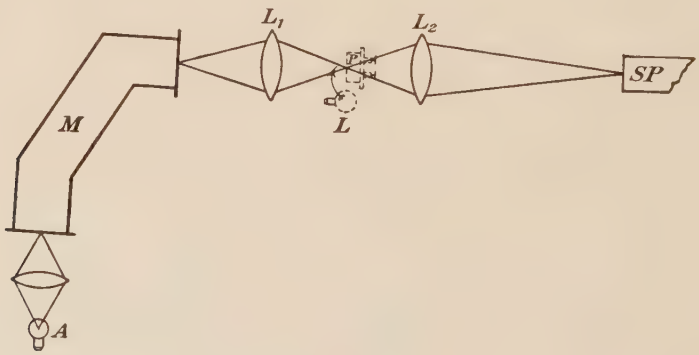

Fig. 10.

The lamp thus standardised may be used for the comparison of distant spectral lines. Fig. 2 shows the Cd triplet with, on the same photographic plate, an image of the standardised lamp.

Now the density curves for the lines as well as for the lamp are constructed. For each wave-length the proportion of the spectral line and the continuous spectrum are determined from the distance of the density curves. And as for the continuous spectrum, the proportion of the intensity for two wave-lengths is known, the pro-

portion of the lines can be determined by this method. The standardised spectrum helps to determine the curve of sensitiveness for the plate.

The above explanations may suffice for a description of our methods. In the following paragraphs I may be permitted to try to give you a short sketch of the results obtained.

As I have mentioned in the beginning, if the intensity is an atomic quality, just like the frequency, then we may expect some simple relationship for the intensity of those spectral lines which are emitted in cases where the electron moves from one energy level to two or more other ones. Such cases occur in the emission of the sharp subordinate series, where the electron falls from the simple s to the multiple p level.

Sommerfeld and Landé have characterised these levels by whole numbers, the so-called internal quantum numbers (j) which are related with the internal magnetic field of the atom. From these internal quantum numbers the statistical weight G of the level can be determined. This weight is proportional to the number of the possible orientations of the path of the electron relative to the trunk of the atom.

Now, from the measurements in the sharp subordinate series there appeared a most remarkable result. For the doublets

Na	1p—3s (6160–6154)	1p—4s (5153–5149)
Ka	1p—4s (5802–5182)	1p—5s (5339–5323)
Rb	1p—4s (6160–6071)	
Cs	1p—5s (6034–5839)	

the proportion of the components appeared to be 2 : 1, with an experimental error of less than 4 per cent.

For the triplets of the earth-alkalis this proportion was found to be 5 : 3 : 1. This result was obtained in my Institute, first for arc-spectra by Dorgelo and afterwards confirmed for the flame spectra of alkalis by Miss Bleeker and Mr. Bongers. Prof. Sommerfeld drew my attention to the fact that the proportion of the intensities for alkalis and earth-alkalis is equal to the proportion of the statistical weight of the final levels. Therefore the triplets of the sharp series of higher multiplicity were examined. The prediction for the intensities was for

quartet system	6 : 4 : 3
quintet	7 : 5 : 3
sextet	8 : 6 : 4
septet	9 : 7 : 5
octet	10 : 8 : 6

The measurement teaches us that, for the triplet of the sextet and the octet system of manganese, the predicted proportion holds true. For Mn λ (6021, 6016, 6013) the ratio found by Dorgelo is 100 : 77 : 53, the ratio expected 100 : 75 : 50 (sextet system); and for λ (4823, 4783, 4754) 100 : 80 : 61 found, and expected 100 : 80 : 60.

Meanwhile, we found that even in the cases where a priori the proportion might possibly depend on the conditions of excitation—i.e., for the cases of the diffuse subordinate series—and also for some cases of the principal series where there is no self-reversion, the proportions do not depend on the circumstances, and may be described by relatively simple whole numbers. Thus there appeared to exist

in those problems of intensity a much profounder regularity than there had even been expected. I will formulate this regularity in the form given to it by my co-operator Dr. Burger, in connexion with the experimental results of Dorgelo described above.

In order to formulate those rules we consider the p–d composed doublet of the alkalis.

	G	Intensities.	
40	4	4	36
20	2	20	0
10		4	6
	6	24	36

In multiplying the sum of the statistical weight of the initial levels (6) with the statistical weights of the final levels, we obtain numbers which are proportional to the energy radiated towards these levels. On the contrary, we obtain numbers proportional to the intensities of the lines starting from the respective initial levels by multiplying the sum of the statistical weights (10) of the final levels with the respective statistical weights of the appropriate initial levels. The diagram gives the calculation for the p–d case (diffuse subordinate series of the alkalis). In the case of Na, Ka, Rb the d-level is not split up. The intensity 36+4 is observed as a whole. In the diffuse series of all those alkalis we have found the proportion of intensities 2 : 1. In the case of Cs, the d-level is sufficiently split up, and we found in accordance with the rule a proportion 9 : 1 : 5. The sum rule has been confirmed by a number of further measurements which were conducted as well in our Institute as by Dr. Dorgelo (at present at Philips Lamp Works Laboratory), and by Dr. Frerichs in the Bonn Institute. The sum rule does not only hold for the spectra of atoms, but also for the ionised atoms in the spark spectra, as recent researches of Dr. H. C. Burger and myself have shown. The same is true for the doublets in a quartet system.

It is worth mentioning that recent measurements in my Institute have shown that for several lines of the principal series of Rb and Cs the ratio of the doublet is 2 : 1, contrary to the older results of Füchtbauer, who measured the ratio of absorption coefficients. The sum rule does not generally suffice to determine all intensities. If we take, for instance, a triplet case, three of the intensities are fixed by the rule that those intensities are zero for which the statistical weight would change with another value than 0 or 2. In the triplet scheme the intensities corresponding to the transitions 5-7 and 1-3 can be fixed directly. Now, if we put the transition 5-3$=x$, the other intensities are fixed by the sum rule, whereas the value of x is left undetermined by this rule

	G	Intensities.		
75	5	x	$12-x$	63
45	3	$12-x$	$33+x$	0
15	7	15	0	0
15		3	5	7
	g	27	45	63

In the same way we may proceed for higher multiplicities. The indefiniteness, however, grows. So far as measurements have been performed the sum rule was always confirmed.

If we take a more general triplet, we obtain for the azimuthal quantum number k the following table.

G	Intensities.		
$2k+1$ $2k-1$ $2k-3$	x $3\cdot4-x$ $3(4k^2-4k-3)$	$3\cdot4-x$ $3(4k^2-5)+x$ 0	$3(4k^2+4k-3)$ 0 0
	$2k-1$	$2k+1$	$2k+3$

It would be easy to give more schemes of this type; however, this one may suffice to show that the proportion of the terms of the diagonal to the terms of the rows parallel to it is for large values of K:

$$K^2 : \text{Const} : \frac{1}{K^2}$$

If we put, with Sommerfeld, $j_a=k-1$, and if j_s* represents the internal quantum number of the s level in the multiplet system, then the constant part of the row next to the diagonal can be represented by

$$2r\{j_s(j_s+1)-(j_e-j_a)(j_a-j+1)\}\ ,\ r=2j_s+1$$

I must draw attention to the fact that considerations developed by Sommerfeld and Heisenberg on the basis of an atomic model and of the Bohr correspondence principle, had led to results which are analogous to those obtained from the sum rule. Therefore Dr. Burger and I have tried to generalise the old Sommerfeld-Heisenberg theory of intensities towards the cases mentioned above. On the basis of our work, Kronig, Sommerfeld and Höhnerl have given other generalisations. However, all these generalisations fail to describe the intensities observed, as measurements on a multiplet of Fe performed in my Institute have shown, so that this part of the problem is still open.

Besides in the intensities in series we have also worked on the intensity problem of the Zeeman effect. It is possible to predict the intensities of the simplest Zeeman effects by simply extending the rules found for the multiplets. Kronig and Höhnerl have generalised our considerations for more complicated cases of the Zeeman effect. Whereas our measurements of our own plates and of plates kindly sent us by Dr. Back in Tübingen confirm the theory. I will describe here our considerations for the p–s triplet.

The splitting up is determined by the so-called magnetic quantum numbers, indicated in the first row. The letters indicate the intensities. $\parallel$ or $\perp$ means the polarisation in the transversal effect. Now we have introduced the following hypotheses:—

1. Every line as a whole is unpolarised.
2. The total energy radiated from the magnetic level to 2, 1, 0, −1, −2, is equal for each of the lines.

* In the notation of Sommerfeld, $s=0$ for the singlet, $\frac{1}{2}$ for the doublet, 1 for the triplet system, and so on.

3. The total energy that reaches the level 1, 0, −1, is equal for each line.

4. In order to determine the relative intensity of the components of all the lines, we use the proportions following from the multiplet theory.

		p_1	s	p_2	s	p_3	s
2	1	a_1					
1	1		C_1		y_1		
1	0	a_2		x_2			
0	1	a_3		x_3		α_3	
0	0		C_2		y_2		β_2
0	−1	a_3		x_3		α_3	
−1	0	a_2		x_2			
−1	−1		C_1		y_1		
−2	−1	a_1					
		⊥	∥	⊥	∥	⊥	∥

The figures a, &c., represent the intensities of the ⊥ and the ∥ components. The given rules lead to

$$a_1+a_2+a_3=C_1+\frac{C_2}{2}=5p \qquad a_1=3p \quad a_2=\frac{3}{2}p \quad a_3=\frac{p}{2}$$

$$a_1=\frac{C_1}{2}+a_2=2a_3+\frac{C_2}{2} \qquad C_1=3p \quad C_2=4p$$

$$a_1+\frac{C_1}{2}+a_3=2a_2+\frac{C_2}{2}$$

$$x_2+x_3=y_1+\frac{y_2}{2}=3p \qquad x_3=x_2=\frac{3}{2}p$$

$$x_2+\frac{y_1}{2}=2x_3+\frac{y_2}{2} \qquad y_2=0$$

$$\frac{y_1}{2}+x_3=2x_2+\frac{y_2}{2} \qquad y_1=3p$$

$$\alpha_3=\frac{\beta_2}{2}=p$$

The experiments have confirmed our calculations.

Besides the theoretical meaning of the sum rule in relation to the correspondence principle, there is still another way to make its contents clear. Let us take, for instance, the case where the radiation is produced by an electron falling from the two levels p_1 and p_2 on one s level. Now the intensity is proportional to the number of atoms at the level, and to a probability of transition. It may, therefore, be represented by

$$N_1A_{p_1s} \text{ resp. } N_2A_{p_2s}$$

or

$$\frac{N_1A_{p_1s}}{4} = \frac{N_2A_{p_2s}}{2}$$

Now N_1 is proportional to the statistical weight G_1 and to a Maxwell-Boltzmann

function $e^{-\epsilon_1/kT}$, where ε_1 represents the energy of the level s. In the same way N_2 is proportional to $G_2 e^{-\epsilon_2/kT}$. As a rule, $\varepsilon_1 - \varepsilon_2$ is small, so that

$$\frac{G_1 A_{p_1 s}}{4} = \frac{G_2 A_{p_2 s}}{2}$$

or

$$A_{p_1 s} = A_{p_2 s}$$

which holds strictly for infinite temperature. The sum rule, therefore, takes this form : *the probability of transition is equal for both p levels.*

Let us now consider the case that the radiation takes place from one level to two others, with the statistical weights 4 and 2, and therefore with a proportion of the intensities 4 : 2.

If A_{sp_1} and A_{sp_2} indicate the transition probabilities, we have the equation

$$\frac{A_{sp_1}}{4} = \frac{A_{sp_2}}{2}$$

However, we will indicate another simpler relation that exists for this case. Einstein in his theory of radiation has introduced the probability of absorption for a unit of radiation density in addition to the probabilities of transition. He calls them

$$B_{p_1 s} \text{ and } B_{p_2 s}$$

Einstein has proved that the existence of the Planck radiation formula requires

$$G_s A_{sp_1} = \frac{8\pi}{c^3} h \nu^3_{sp_1} G_1 B_{p_1 s}$$

and

$$G_s A_{sp_2} = \frac{8\pi}{c^3} h \nu^3_{sp_2} G_2 B_{p_2 s}$$

where c is the velocity of light, h Planck's quantum and ν_{sp_1}, &c., the frequencies of the lines considered. From this we get

$$\frac{A_{sp_1}}{A_{sp_2}} = \frac{G_1}{G_2} \cdot \frac{B_{p_1 s} \nu^3_{sp_1}}{B_{p_2 s} \nu^3_{sp_2}}$$

or with the relation for $\dfrac{A_{sp_1}}{A_{sp_2}} = \dfrac{4}{2}$

$$\nu^3_{p_1 s} B_{p_1 s} = \nu^3_{p_2 s} B_{p_2 s}$$

For a more complicated case we find

$$G_7 A_{75} + G_5 A_{55} + G_3 A_{35} :$$
$$G_5 A_{53} + G_3 A_{33} :$$
$$G_3 A_{31} \therefore$$
$$G_5 : G_3 : G_1$$

Now we have by the Einstein relation

$$G_7 A_{75} = \frac{8\pi}{c^3} \nu^3{}_{75} B_{75} G_5$$

therefore

$$\nu^3{}_{57} B_{57} + \nu^3{}_{55} B_{55} + \nu^3{}_{35} B_{53} = \nu^3{}_{53} B_{35} + \nu^3{}_{33} B_{33} = \nu^3{}_{31} B_{13}$$

and also

$$G_7 A_{75} : G_5 (A_{55} + A_{53}) : G_3 (A_{35} + A_{33} + A_{31}) :: G_7 : G_5 : G_3$$

or

$$A_{75} = A_{55} + A_{53} = A_{35} + A_{33} + A_{31}$$

which can be generalised easily.

It may be of great importance to remark that the quantity $\nu^3{}_{p_1 s} B_{p_1 s}$ can be reduced to a universal quantity if we adopt the light quantum hypothesis. Einstein assumes that for a radiation density the probability of absorption can be represented by $B\rho$. If we postulate light to consist of quanta $h\nu$, then $\rho = n h\nu$, where n is the number of quanta per unit of volume. Burger and I have proved that the probability for a quantum to strike an atom is proportional to λ^2, where λ represents

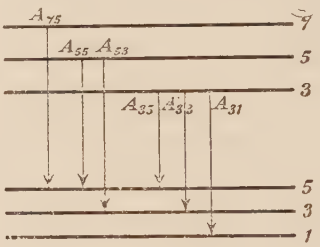

FIG. 11.

the wave-length. The number of quanta that strike an atom per unit of time, and that are absorbed, can be represented, therefore, by $C\lambda^2 n$, where C is a constant depending on the character of the atom.

Now we have

$$n h\nu B = C n \lambda^2$$

or

$$B\nu^3 = C$$

Therefore we obtain the rule that the sum of the absorption probabilities per energy-quantum is the same for all the levels of a multiplet.

The fact that there exist regularities of intensity to such a degree makes it desirable to extend research to other cases also. In the Röntgen region similar rules have been established.

At present I may mention some other problems which have been partly solved. It will be known that in many cases the intensity stongly depends on the conditions of excitation. For instance, if electrons of given velocities are sent into Na vapour, it will be possible to excite the D lines separately. Still, it is possible to define a spectrum that is normal as to the intensities, this being the spectrum emitted

at very high temperature. At such a temperature every state of the atom will be present proportional to the statistical weight, and it will emit proportionally to its inherent probability. To approximate this spectrum theoretically as well as experimentally will be the intensity problem of the future.

There are some experiments which perhaps show the existence of such a spectrum. They relate to observations of the intensity in flames and the arc of the subordinate series of the alkalis Na, Ka, Rb, Cs. From the work of Miss Bleeker and Mr. Bongers it appears that the intensity of the terms of the series as a function of the ordinal number is independent of temperature, the temperature having only a secondary influence on the total intensity of the flame. A discussion of these results will appear shortly on the base of the correspondence principle. In a flame the atoms are ionised by a chemical process, and then send out the normal spectrum.

Finally, I should like to mention the possibility of determining the energy of the levels by means of intensity measurement. The experiments have not yet been finished, but the provisional results already show this possibility.

We have determined the change of several spectral lines of the mercury arc as a function of the energy put into the arc. Lines emitted by the electron when starting from levels for which the energies differ but little have the same dependance on the energy put into the arc. If sufficient material be obtained, the slopes of the curves energy put in the arc energy of the level, expressed as a function of the latter, will allow us to calculate the energy of unknown levels.

I hope that my lecture has contributed to showing the importance of quantitative measurement of intensities, and that I may have succeeded in pointing out to you that the intensity of spectral lines represents an atomic quality as important for our knowledge of the atom as are the frequencies of radiation of their Zeeman-effect.

Permit me, in conclusion, to thank you most cordially for the opportunity you have given me to expound these problems to your Society.

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DEMONSTRATION OF SOME SIMPLE APPARATUS FOR THE ESTIMATION OF CARBON DIOXIDE.

By Dr. Ezer Griffiths, of the National Physical Laboratory.

APPLES respire, generating carbon dioxide. It has been found that the atmosphere of a gas-tight chamber gradually becomes richer in carbon dioxide and deficient in oxygen, and when the carbon-dioxide content exceeds a certain value the apples are liable to become diseased. The instruments demonstrated were devised so as to provide marine engineers with simple means of making analyses of the atmosphere in the ship's holds carrying an apple cargo.

One was of the simple gas burette type, in which a measured volume of the gas under test is displaced into a potash bulb and then returned to the burette for measurement after absorption of the carbon dioxide. Float valves are provided to prevent potash solution being accidentally drawn back into the burette or water swept over into the potash.

The novel feature of the second was a large hollow-barrel glass tap. Absorbent for carbon dioxide was packed into this hollow barrel. In one position of the tap a sample of gas was pumped through into a tube ; in the second position the absorbent was in communication with the sample of gas resulting in an absorption of the carbon dioxide and consequent diminution of volume of the sample which was measured.

The third instrument was a large-bore tube bent into a nearly complete circle. This tube was mounted on an axis through the centre of the circle, so that it could be partially rotated. A large drop of mercury functioned as a piston for displacing the sample of gas contained in the tube through an absorbent cartridge. The diminution in pressure was indicated on a dial gauge calibrated to read in carbon-dioxide percentages.

The fourth instrument was a specially-designed brass pump, by means of which a sample of gas is withdrawn from the ship's hold, and then displaced through an absorbent cartridge. As in the previous instrument, the carbon-dioxide content is indicated on a dial gauge.

DEMONSTRATION OF SOME EXPERIMENTS TO ILLUSTRATE THE APPLICATION OF THE SHAKESPEARE KATHAROMETER TO PHYSICAL RESEARCH.

By Dr. H. L. Daynes, of the Cambridge Scientific Instrument Co.

THE following characteristics of the instrument were pointed out :—

(1) A direct reading is given without the withdrawal of any gas for analysis or disturbance of the conditions of the experiment.

(2) The readings are very rapid. The half-value-period for hydrogen-air mixtures is of the order of 10 seconds, and is longer for gases of higher molecular weight. The actual relation between the periods for different gases is calculable from the diffusion constant for the mixture considered.

(3) The capacity is small, being only about 0·5 c.c. Small quantities of gas, therefore, give large changes of percentage in the mixture if the system to which the instrument is connected is also small.

(4) The instrument is sensitive to small changes of composition. With a reflecting galvanometer a charge of one part of hydrogen in 100,000 of air can be detected if the change takes place not so slowly that zero changes mask the deflection.

(5) The instrument is essentially a differential one, and has all the advantages of the differential method in eliminating the effects of variations other than the particular one to be studied. The full sensitivity of the method may, therefore, be utilised in a great variety of experiments.

In illustration of the above points, the following series of experiments was shown :—

(1) Measurement of a small rate of flow of gas by addition of hydrogen at a known rate from an electrolytic cell, and measurement of the resulting percentage of hydrogen.

(2) The rise in CO_2 percentage in an enclosed space due to the respiration of a small bee. A few thousandths of a cubic centimetre were detectable.

(3) The leakage of CO_2 and H_2 from a few square millimetres of two rubber balloons was visible in a few seconds.

(4) A method of demonstrating the effect of "thermal diffusion" in producing separation of hydrogen from oxygen. The method gives rapid results, which can easily be interpreted in absolute units to give values of the "Chapman" constant for a mixture. A simple method of calibrating is included in the apparatus.

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